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# Modelling The Joint Dynamics of Rail Vikas And Indian Railway Finance Corporation (IRFC) Using A Multivariate Markov Process

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## ABSTRACT

In time series analysis, Markov models are frequently employed methods. A stochastic method for modeling systems in which the future state solely depends on the present state is the Markov Chain (MC). By adding more than one variable, the Multivariate Markov Model (MMC) expands on the classic Markov model to examine the likelihood of future states. The relationship between the stock prices of IRFC and Rail Vikas is investigated in this study, along with the ways in which each stock price affects the other and the likelihood of price movements. The MMC model is used to determine whether these stocks tend to move in tandem and to predict whether they will rise, fall, or stay the same. The study emphasizes how the stock prices of Rail Vikas and IRFC, two companies with major roles in the transportation and infrastructure industries, are actively correlated. The MMC model successfully captures non-linear relationships and regime shifts that conventional models like linear regression and VAR might not adequately account for when using historical stock data. The results show that there are times when the two stocks are correlated, especially when the market is moving. The study concludes that MMC offers insightful information about the short- and long-term relationships between the stock prices of IRFC and Rail Vikas, with useful ramifications for investors and decision-makers who want to comprehend market dynamics.

*Keywords: Multivariate Markov chain, Markov model, linear regression, Vector Auto Regressive, Stochastic process*

## INTRODUCTION

Stochastic modelling plays a pivotal role in addressing the complexities of forecasting and decision-making in dynamic environments. In the context of Indian Railways, the Indian Railways Finance Corporation (IRFC) under the Ministry of Railways serves as a crucial financial arm by meeting the ‘Extra Budgetary Resources’ (EBR) requirements through market borrowing. Since its inception, IRFC has financed over ₹4 lakh crores, contributing to rolling stock procurement, infrastructure development, and capacity-building projects, ensuring sustainable growth in the railway sector. Publicly listed on the National Stock Exchange (NSE) and Bombay Stock Exchange (BSE) since January 29, 2021, IRFC embodies a significant transformation from a public limited company to a recognized Non-Banking Financial Company (NBFC). Its adherence to prudential norms supports its ongoing efforts to finance railway entities like Rail Vikas Nigam Limited (RVNL), RailTel, Konkan Railway Corporation Limited (KRCL), and Pipavav Railway Corporation Limited (PRCL). The growing interest of traders and investors in government stocks like IRFC is driving greater infrastructure investments, with technical and statistical analyses guiding stock price forecasting. Stochastic processes, categorized by the characteristics of their state space, index parameters, and random variable dependencies, are integral to these forecasting models. Markov processes (MP), where the future state depends solely on the present, offer a powerful framework for analysing complex systems. A discrete state space MP, known as a Markov Chain (MC), has been widely applied in financial and commodity markets. For instance, Vivian & Wohar (2021) utilized a Multivariate Markov Chain (MMC) model to analyse the co-movement of oil and gold prices during economic uncertainty, demonstrating that investors hedge against inflation by investing in gold when oil prices rise. Deng & Huang (2021) extended this approach by incorporating external indicators like inflation rates and currency exchange rates into a time-varying MMC, achieving superior forecasting accuracy, especially in volatile markets. This study integrates these concepts by applying an MMC model to forecast the dynamic relationship between Stock price of IRFC and Rail vikas period between January 2021 and July 2022. Using inter-transition and intra-transition matrices, the model captures the influence of economic factors on price behaviour. By combining stochastic modelling techniques across infrastructure finance and commodity forecasting, this paper underscores the versatility of these models in addressing the uncertainties of diverse domains. For IRFC, stochastic models provide valuable insights into stock price behaviour, guiding strategic investments in India's transportation infrastructure. Similarly, the application of MMC to commodity prices demonstrates the robustness of stochastic methods in predicting and managing market dynamics, facilitating informed decision-making across industries.

## REVIEW OF LITERATURE

**James Hamilton in (1989)** explores the evolution and application of Markov switching models in econometrics. These models allow for regime changes in time series data, effectively capturing shifts in economic conditions such as transitions between growth and periods. **Ba, J., Del Campo, C., & Kelle, L. R. (2017)** compare the two software's future state probabilities. The software most used in medical applications is produced by Tree Age, since it offers many advantages to the user. **Ghosh et al. (2022)** applied a multivariate hidden Markov model (HMM) to the oil-gold market, focusing on the latent states driving the relationship between the two. **Hafez & Rizwan (2022)** explored how geopolitical risks have increased the co-movement of crude oil and gold prices using a multivariate regime-switching Markov model. **Almutiri, T., & Nadeem, F. (2022)** MC, which create a categorization of words to make a complete sentence, is often used in generating natural language. It reviews Markov models' use in three applications of natural language processing (NLP): natural language generation, named-entity recognition, and parts of speech tagging. **Tiwari et al. (2020)** employ a Markov-switching time-varying copula model to explore the dependency between gold and oil prices, highlighting the influence of geopolitical risks on this relationship. **Arif et al. (2022)** introduce a long memory model combined with a fuzzy time series Markov Chain to analyze precious metal price movements, emphasizing the persistence of shocks in gold prices. **Guan et al. (2021)** utilize a Markov switching VAR approach to investigate the drivers of oil prices, offering insights into regime changes and their impact on market dynamics.

## METHODOLOGY

The sequence $\{X_n, n \geq 0\}$ is said to be a MC for all state prices $i_0, i_1, i_2, \dots, i_n \epsilon I$, then,

$$P\{ X_{n+1} = j | X_0 = i_0, X_1 = i_1, \dots, X_n = i\} = P\{X_{n+1} = j | X_n = i\}$$

where, $i_0, i_1, i_2, \ldots, i_n$ are the states in the state space *I*. This type probability is MC probability. This show that nevertheless of its history prior to time n, the probability that will make a changeover to another state *j* depends only on state *i*.

## Multivariate Markov Chain

In MMC model represents the behaviour of multiple categorical sequences generated by alike sources. Here it is assumed that there are "s" categorical sequences and each has "m" possible states in M. Let $x_n^{(k)}$ be state vector of the k$^{th}$ sequence at time n. If the k$^{th}$ sequence is in state j at time n then

$$x_n^{(k)} = ej = (0, \ldots 0,1,0, \ldots ,0)^T$$

It is assumed that there is the following relationship:

$$x_{n+1}^{(j)} = \sum_{k=1}^{s} \lambda_{jk} P^{(jk)} x_n^{(k)}, for\ j = 1,2\ldots,s$$

Where $\lambda_{jk} \geq 0, 1 \leq j,k \leq s\ and \sum_{k=1}^{s} \lambda_{jk}, for\ j = 1,2 \ldots s.$

The state probability distribution of the k$^{th}$ sequence at the (n+1)$^{th}$ step depends on the weighted average of $P^{(jk)} x_n^{(k)}$. Here $P^{(jk)}$ is a Transition Probability Matrix (TPM) from the states in the k$^{th}$ sequence to the states in the j$^{th}$ sequence, and $x_n^{(k)}$ is the state probability distribution of the kth sequences at the nth step. In matrix form it is written:

$$\boldsymbol{x_{n+1}} = \begin{bmatrix} x_{n+1}^{(1)} \\ x_{n+1}^{(2)} \\ \vdots \\ x_{n+1}^{(s)} \end{bmatrix} = \begin{bmatrix} \lambda_{11} P^{(11)} & \lambda_{12} P^{(12)} & \cdots & \lambda_{1s} P^{(1s)} \\ \lambda_{21} P^{(21)} & \lambda_{22} P^{(22)} & \cdots & \lambda_{2s} P^{(2s)} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{s1} P^{(s1)} & \lambda_{s2} P^{(s2)} & \cdots & \lambda_{ss} P^{(ss)} \end{bmatrix} \begin{bmatrix} x_n^{(1)} \\ x_n^{(2)} \\ \vdots \\ x_n^{(s)} \end{bmatrix} \equiv \mathrm{Q} x_n$$

It is proposed that some approaches for the estimations of $P^{(jk)}$ and $\lambda_{jk}$. For each data sequence, the TPM is estimated by the following method. Given the data sequence, the transition frequency from the states in the k$^{th}$ sequence to the states in the j$^{th}$ sequence is counted. Hence, the transition frequency matrix for the data sequence can be constructed. After the normalization, the estimates of the TPM for the MMC model has to be estimated. More precisely, the transition frequency $f_{i_j i_k}^{(jk)}$ from the state $i_k$ in the sequence $\{x_n^{(k)}\}$ to the state $i_j$ in the sequence $\{x_n^{(j)}\}$ is counted and therefore the transition frequency matrix for the sequences is constructed as follows:

$$F^{(jk)} = \begin{bmatrix} f_{11}^{(jk)} & f_{21}^{(jk)} & \cdots & f_{m1}^{(jk)} \\ f_{12}^{(jk)} & f_{22}^{(jk)} & \cdots & f_{m2}^{(jk)} \\ \vdots & \vdots & \vdots & \vdots \\ f_{1m}^{(jk)} & f_{2m}^{(jk)} & \cdots & f_{mm}^{(jk)} \end{bmatrix}$$

From $F^{(jk)}$ , the estimates for $P^{(jk)}$ are obtained as follows:

$$\hat{P}^{(jk)} = \begin{bmatrix} \hat{P}_{11}^{(jk)} & \hat{P}_{21}^{(jk)} & \cdots & \hat{P}_{m1}^{(jk)} \\ \hat{P}_{12}^{(jk)} & \hat{P}_{22}^{(jk)} & \cdots & \hat{P}_{m2}^{(jk)} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{P}_{1m}^{(jk)} & \hat{P}_{2m}^{(jk)} & \cdots & \hat{P}_{mm}^{(jk)} \end{bmatrix}$$

Where,

$$\hat{P}^{(jk)} = \begin{cases} \dfrac{f_{i_j i_k}^{(jk)}}{\sum_{i_k=1}^{m} f_{i_j i_k}^{(jk)}} & if\ \sum_{i_k=1}^{m} f_{i_j i_k}^{(jk)} \neq 0 \\ 0 & otherwise \end{cases}$$

## RESULTS

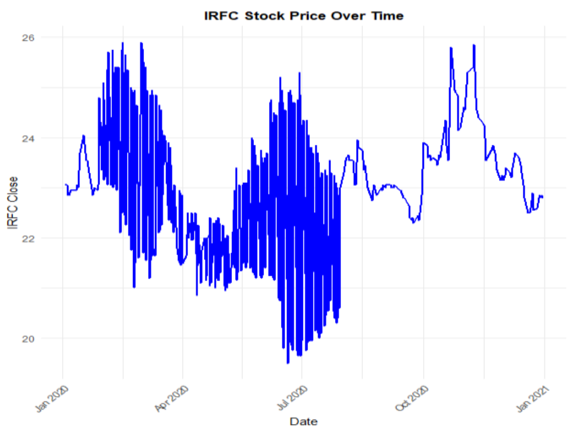


***Figure 1. Line plot for the IRFC Stock***

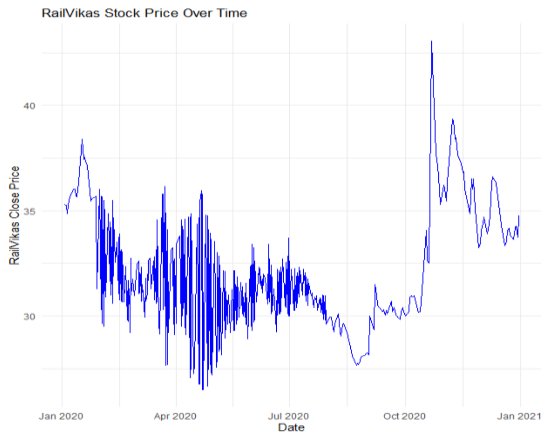


***Figure 2. Line plot for the RailVikas Stock***

***Table 1. Relationship between petrol and gold price in Chennai market***

| | Railvikas | IRFC |
|---|---|---|
| **Railvikas** | 1 | 0.2905806 |
| **IRFC** | 0.2905806 | 1 |

Coefficient is 0.89512. the result signifies that the two variables such as IRFC and Railvikas stocks' close price are moderately correlated with each other.

The intra transition probability matrix of IRFC price can be constructed as

$$P_{IRFC} = \begin{bmatrix} 0.5080 & 0.3636 & 0.1283 \\ 0.5138 & 0.4236 & 0.0625 \\ 0.4500 & 0.3750 & 0.1750 \end{bmatrix}$$

The intra transition probability matrix of Railvikas price can be constructed as

$$P_{Railvikas} = \begin{bmatrix} 0.5549 & 0.4188 & 0.0261 \\ 0.4702 & 0.4821 & 0.0476 \\ 0.4166 & 0.5833 & 0.0000 \end{bmatrix}$$

The inter transition probability matrix of IRFC to Railvikas price can be constructed as

$$P_{IRFC*Railvikas} = \begin{bmatrix} 0.6791 & 0.2994 & 0.2153 \\ 0.3172 & 0.6275 & 0.0551 \\ 0.4500 & 0.5250 & 0.0250 \end{bmatrix}$$

The inter transition probability matrix of Railvikas to IRFC price can be constricted as

$$P_{Railvikas\,*\,IRFC} = \begin{bmatrix} 0.6649 & 0.2408 & 0.0942 \\ 0.3333 & 0.5416 & 0.1250 \\ 0.3076 & 0.6153 & 0.0769 \end{bmatrix}$$

**Determination of initial state vector for IRFC price**

The initial state vector of IRFC price can be constructed as

$$\eta_{IRFC_1} = \frac{187}{371} = 0.50404$$

$$\eta_{IRFC_2} = \frac{144}{371} = 0.3881$$

$$\eta_{IRFC_3} = \frac{40}{371} = 0.1078$$

The initial state vector for IRFC price,

$$\eta_{IRFC}\ (0) = (0.5040 \quad 0.3881 \quad 0.1078)$$

**Determination of initial state vector for Railvikas price**

The initial state vector of Railvikas price can be constructed as

$$\eta_{Railvikas_1} = \frac{190}{371} = 0.5148$$

$$\eta_{Railvikas_2} = \frac{168}{371} = 0.4528$$

$$\eta_{Railvikas_3} = \frac{13}{371} = 0.0350$$

The initial state vector for Railvikas price,

$$\eta_{Railvikas}\ (0) = (0.5148 \quad 0.4528 \quad 0.0323)$$

**Determination of initial state vector for IRFC to Railvikas price**

The initial state vector of IRFC to Railvikas price can be constructed as

$$\eta_{IRFC*\,Railcikas_1} = \frac{187}{372} = 0.5026$$

$$\eta_{IRFC*\,Railcikas_2} = \frac{145}{372} = 0.3897$$

$$\eta_{IRFC*\,Railcikas_3} = \frac{40}{372} = 0.1075$$

The initial state vector for IRFC to railvikas price,

$$\eta_{IRFC*\,Railcikas}\,(0) = (0.5026 \quad 0.3897 \quad 0.1075)$$

**Determination of initial state vector for Railvikas to IRFC Price**

The initial state vector of Railvikas to IRFC price can be constructed as

$$\eta_{Railcikas*IRFC_1} = \frac{191}{372} = 0.5134$$

$$\eta_{Railcikas*IRFC_2} = \frac{168}{372} = 0.4516$$

$$\eta_{Railcikas*IRFC_3} = \frac{13}{372} = 0.0349$$

The initial state vector for Railvikas to IRFC price,

$$\eta(0) = (0.5134 \quad 0.4516 \quad 0.0349)$$

**Long Term Prediction for IRFC price**

$$\eta_{IRFC}\,(0) * P^1 = (0.5040 \quad 0.3881 \quad 0.1078) \begin{bmatrix} 0.5040 & 0.3881 & 0.1078 \\ 0.5040 & 0.3881 & 0.1078 \\ 0.5040 & 0.3881 & 0.1078 \end{bmatrix}$$

$$= (0.5040 \quad 0.3881 \quad 0.1078)$$

**Long Term Prediction for Railvikas Price**

$$\eta_{Railvikas}\,(0) * P^2 = (0.5148 \quad 0.4528 \quad 0.0323) \begin{bmatrix} 0.5117 & 0.4532 & 0.0349 \\ 0.5117 & 0.4532 & 0.0349 \\ 0.5117 & 0.4532 & 0.0349 \end{bmatrix}$$

$$= (0.5117 \quad 0.4532 \quad 0.0349)$$

**Long Term Prediction for IRFC to Railvikas Price**

$$\eta_{IRFC*Railvikas}(0) * P^3 = (0.5026 \quad 0.3897 \quad 0.1075) \begin{bmatrix} 0.5048 & 0.4581 & 0.0370 \\ 0.5048 & 0.4581 & 0.0370 \\ 0.5048 & 0.4581 & 0.0370 \end{bmatrix}$$

$$= (0.5048 \quad 0.4581 \quad 0.0370)$$

**Long Term Prediction for Railvikas to IRFC Price**

$$\eta_{Railvikas*IRFC}(0) * P^4 = (0.5134 \quad 0.4516 \quad 0.0349) \begin{bmatrix} 0.4946 & 0.4005 & 0.1047 \\ 0.4946 & 0.4005 & 0.1047 \\ 0.4946 & 0.4005 & 0.1047 \end{bmatrix}$$

$$= (0.4946 \quad 0.4005 \quad 0.1047)$$

## DISCUSSION

In brief, the first variate (IRFC) affects the second variate (Railvikas) and this correlation between these variables help to find the next state by using the MMC. Finding the correlation between the two variables (IRFC and Railvikas) and they moderately correlated then moving for the MMC the MP tells future states depend on current state MMC shows future states depends not only of first state it also depends on second state. By using these properties finding the intra-transition and inter transition matrix and short-term prediction shows the next state step by step and in long run behaviour makes it a steady state and predict the next step to long term. Previous study demonstrates the various future states depend on the increasing, decreasing, or remains rest within the single state this study illustrates that one state increasing, decreasing or resting position will affect the other state.

## CONCLUSION

IRFC Stock Price Forecast (Long-Term):

- Based on the transition probabilities and the initial state vector of Decrease:50.40, Increase: 38.81%, Same: 10.78% the steady state probabilities suggest that IRFC stock prices will decrease with a probability of 50.40%, increase with a probability of 38.81%, and remain the same with a likelihood of 10.78%.

RailVikas Stock Price Forecast (Long-Term):

- Starting from an initial distribution of Decrease:51.48, Increase: 45.28%, Same: 3.23%, the RailVikas stock is expected to decrease with a probability of 51.17%, increase with a probability of 45.32%, and remain the same with a probability of 3.49%. This points to a nearly balanced but slightly bearish trend.

IRFC to RailVikas Cross-Transition Forecast:

- With IRFC as the originating stock and an initial state vector of Decrease:50.27, Increase: 38.98%, Same: 10.75%, the long-term forecast indicates that RailVikas will be in a decreasing state with a probability of 50.48%, increasing with a probability of 45.81%, and remaining the same with a probability of 3.70%. This suggests a mildly bullish influence of IRFC price changes on RailVikas stock over time.

RailVikas to IRFC Cross-Transition Forecast:

- Beginning from an initial distribution of Decrease:51.34, Increase: 45.16%, Same: 3.49%
  the model estimates that IRFC will be in a decreasing state with a probability of 49.46%, increasing with a probability of 40.05%, and remaining the same with a probability of 10.47%. This implies a slightly negative long-term impact of RailVikas price behaviour on IRFC.

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