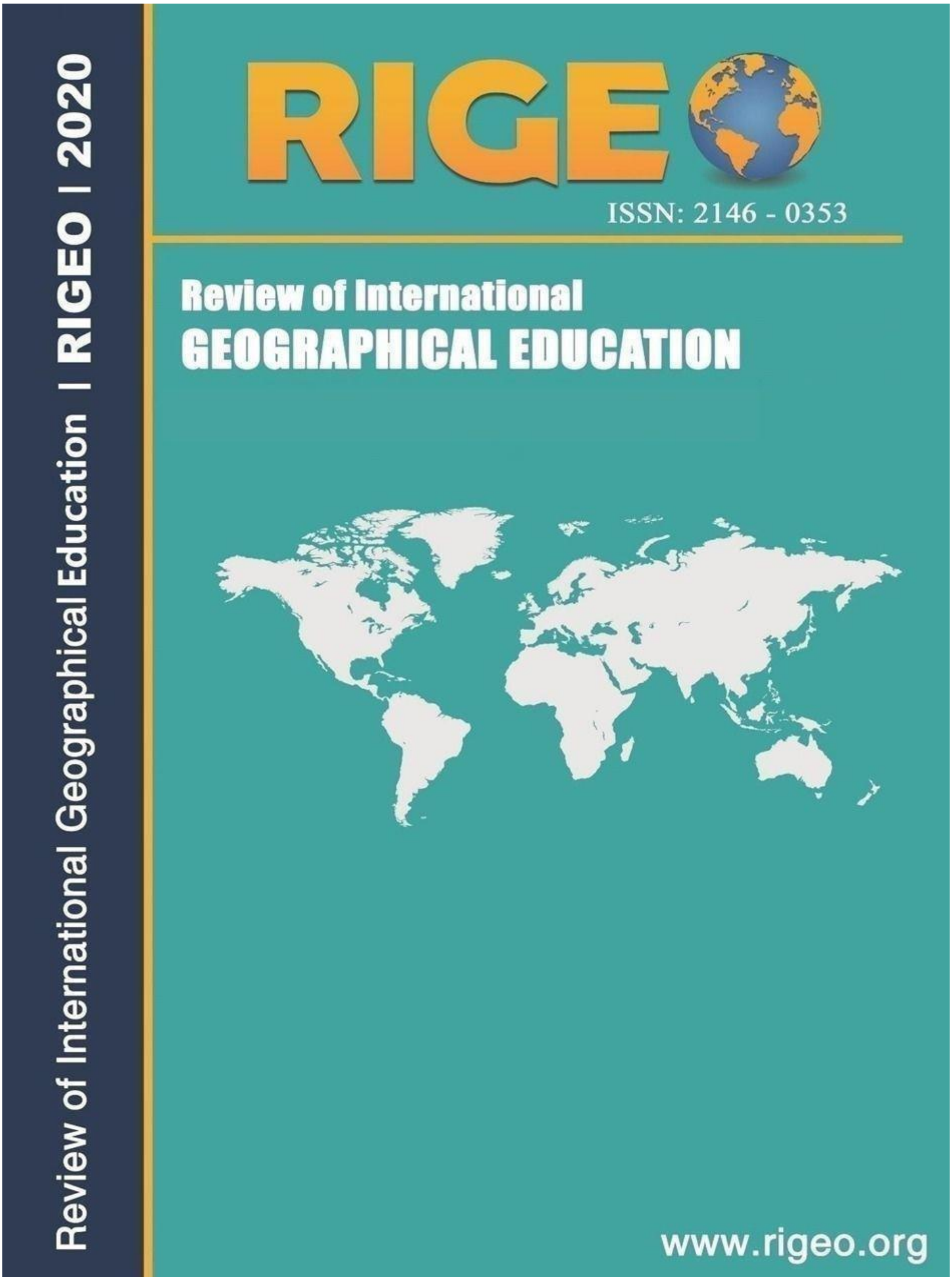
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Cyclic Codes for MDS and MHDR on Finite Chain Rings

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1. Introduction

Coding theory aims to provide optimal codes for detecting and correcting a maximum number of errors during data transmission through noisy channels. Cyclic codes have been in focus due to their rich algebraic structure which enables easy encoding and decoding of data through the process of channel coding. Cyclic codes over rings have gained a lot of importance after the remarkable break- through given by Hammons et al. in reference [1]. A vast literature is available on the structure of cyclic codes overfields, integer residue rings, Galois rings, finite chain rings, and some finite nonchain rings [2–29]. Cyclic codes overfinite chain rings with length coprime to the characteristic ofresidue field have been investigated in references [2, 16, 22]. Islam and Prakash have established a unique set of gener- ators for cyclic codes over Z_{p^k} in reference [4] and for cycliccodes over $F_q \quad uF_q, u^2 \quad 0$ in reference [5]. A. Sharma and T. Sidana have studied cyclic codes of p^s length over finite chain rings in reference [15], thereby extending the results ofKiah et al. on cyclic codes over Galois rings [14]. Dinh explored the structure and properties of cyclic codes oflength p^s over finite chain rings with nilpotency index 2 [13]. However, in most of the studies, there have been some limitations on either the length of code or the nilpotency index of the ring. We do not impose any such restriction in this paper. Salagean made use of the existence of a Grobnerbasis for an ideal of a polynomial ring to establish a unique set of generators for a cyclic code over a finite chain ring with arbitrary parameters [18]. Al-Ashker et al. have also workedin the same direction in the paper [28] by extending the novel approach given by Siap and Abualrub [12] which pulls back the generators of a cyclic code over Z_2 to establish the structure of cyclic codes over the ring $Z_2 \quad uZ_2$ $+ \quad + \cdots + \quad u^{k-1} Z_{k\,1}, u^k\, 0$. They have also extended this approach over thefinite chain ring $F_q \quad uF_q \quad + \quad + \cdots + \quad u^{k-1}F_{q\;1}, \; u^k \quad 0$ [24]. Monika and Sehmi have given a constructive approach to establish a generating set for a cyclic code over a finite chain ring by making use of minimal degree polynomials of certain subsets of the code [20]. We make some advance- ments to this study by establishing a unique set of generatorsfor a cyclic code over a finite chain ring with arbitrary parameters. It is noted that this unique set of generators retains all the properties of generators obtained in reference [20].

The paper is organised as follows: In Section 2, we statesome preliminary results. In Section 3, we establish a uniqueset of generators for a cyclic code over a finite chain ring. InSection 4, we establish a minimal spanning set and rank of the cyclic code. We give sufficient as well as necessary conditions for a cyclic code to be an MDS code. We establish sufficient as well as necessary conditions for a cyclic code oflength which is not coprime to the characteristic of residue field of the ring to be an MHDR code. Finally, we providea few examples of MDS and MHDR cyclic codes over somefinite chain rings.

2. Preliminaries

Let R be a finite commutative chain ring. Let c be the unique maximal ideal of R and] be the nilpotency index of c. Let F_q R/c be the residue field of R, where q p^s fora prime p and a positive integer s.

The following is a well-known result (for reference, see [15]).

Proposition 1. *Let R be a finite commutative chain ring.Then, we have the following:*

3. Unique Set of Generators

In this section, a unique set of generators for a cyclic code $C$of arbitrary length n over R has been established. For this, letus first recall the construction given by Monika et al. to obtain a generating set for a cyclic code C over a finite chainring R [20]. Let $f_0(z), f_1(z), \ldots, f_m(z)$ be minimal degree polynomials of certain subsets of C such that $\deg(f_j(z))$ � $t_j < n$ and the leading coefficient of $f_j(z)$ is equal to $c^{i_j} u_j$, where u_j is some unit in R, $t_j < t_{j+1}$, $i_j > i_{j+1}$, and i_j is the smallest of such power. If i_0 � 0, then $f_0(z)$ is a monic

(i) char$R$ � p^a, where $1 \le a \le$] and $|R|$ � $|F_q|^]$ � $p^{s]}$

polynomial and we have m � 0.

(ii) There exists an element $\zeta \in R$ with multiplicative order $p^s - 1$. The set T � 0, 1, $\zeta, \zeta^2, \ldots, \zeta^{p^s-2}$ is called the Teichmüller set of R

(iii) Every $r \in R$ can be uniquely expressed as r � $r_0 + r_1 c + \cdots + r_{]-1} c^{]-1}$, where $r_i \in$ T for $0 \le i \le] - 1$. Also, r is a unit in R if and only if $r \neq 0$

Lemma 3 (see [20]). *Let C be a cyclic code having a length $n$over R and $f_j(z)$, $0 \le j \le m$, be polynomials as defined above. Then, we have the following:*

(i) C is generated by the set $f_j(z)$; j � $0, 1, \ldots, m$

(ii) For $0 \le j \le m$, $f_j(z)$ � $c^{i_j} h_j(z)$, where $h_j(z)$ is

$$+ \alpha_{1,0} u_1(z) + \alpha_{1,1} z u_1(z) + \cdots + \alpha_{1,t_2-t_1-1} z^{t_2-t_1-1} u_1(z)$$

$$+ \alpha_{0,0} u_0(z) + \alpha_{0,1} z u_0(z) + \cdots + \alpha_{0,t_1-t_0-1} z^{t_1-t_0-1} u_0(z).$$

This implies that $z^{n-t_m-1} u_m(z)$ � $\alpha_m(z) u_m(z) + \alpha_{m-1}(z) u_{m-1}(z) + \cdots + \alpha_0(z) u_0(z)$, where $\alpha_m(z)$ � $\alpha_{m,0} + \alpha_{m,1} z + \cdots + \alpha_{m,n-t_m-2} z^{n-t_m-2}$ and $\alpha_i(z)$ � $\alpha_{i,0} + \alpha_{i,1} z + \cdots + \alpha_{i,t_{i+1}-t_i-1} z^{t_{i+1}-t_i-1}$ for $0 \le i \le m-1$. Clearly, $\deg(\alpha_m(z)) \le n - t_m - 2$ and $\deg(\alpha_i(z)) \le t_{i+1} - t_i - 1$ for all i, $0 \le i \le m-1$. Then, bymultiplying equation (18) by $c^{]-i_{m-1}}$, we get

independent, and hence, it is a minimal spanning set for C. Itfollows that rank (C) � $n - t_0$.

The following theorem determines all the MDS cyclic codes of arbitrary length over a finite chain ring R. □

4. MDS and MHDR Cyclic Codes over a FiniteChain Ring

In this section, the minimal spanning set and rank of a cycliccode C over a finite chain ring R have been established. Sufficient as well as necessary conditions for a cyclic code tobe an MDS code and for a cyclic code to be an MHDR codehave been obtained. Finally, to support our results, some examples of optimal cyclic codes have been presented.

Theorem 1. *Let C be a cyclic code having an arbitrary length n over a finite chain ring R. Then, rank(C) � $n - t_0$,*

this by induction on j. First, we prove that $z^{t_1-t_0} u_0(z) \in$ span S'. Clearly, $z^{t_1-t_0} u_0(z)$ is a polynomial ofdegree t_1 in C. Then, we have $z^{t_1-t_0} u_0(z) - c^{i_0-i_1} u_1(z)$ �

$q_0(z)u_0(z)$ for some $q_0(z) \in R[z]$ with a degree less than $t_1 - t_0$ which implies that $z^{t_1-t_0}u_0(z) - c^{i_0-i_1}u_1(z) \in$ span S'. Therefore, we have $z^{t_1-t_0}u_0(z) \in$ span S'. We suppose that $z^{t_2-t_1}u_1(z), z^{t_3-t_2}u_2(z), \ldots, z^{t_j-t_{j-1}}u_{j-1}(z) \in$ span S' for $1 \le j \le m-1$. Now, we will show that $z^{t_{j+1}-t_j}u_j(z) \in$ span S'. Clearly, $z^{t_{j+1}-t_j}u_j(z)$ is a polynomial of degree t_{j+1} in C. Then, we have $z^{t_{j+1}-t_j}u_j(z) - c^{i_j-i_{j+1}}u_{j+1}(z) \in \langle u_0(z), u_1(z), \ldots, u_j(z)\rangle$; and $z^{t_{j+1}-t_j}u_j(z) = c^{i_j-i_{j+1}}u_{j+1}(z) + m_0(z)u_0(z) + m_1u_1(z) + \cdots + m_ju_j(z)$, where $m_i(z) \in R[z]$ and $\deg(m_i(z)) < t_{i+1} - t_i$ for all i, $0 \le i \le j$. This implies that

where t_0 is the degree of minimal degree polynomial in C.

$m_iu_i(z) \in$ span S' for $0 \le i \le j$, which further implies that $z^{t_{j+1}-t_j}u(z) \in$ span S'. Therefore, we have $z^{t_{j+1}-t_j}u(z)$

Proof. Let C be a cyclic code having an arbitrary length n

$$
\begin{aligned}
z^{n-t_m-1}u_m(z) = {} & \alpha_{m,0}u_m(z) + \alpha_{m,1}zu_m(z) + \cdots + \alpha_{m,n-t_m-2}z^{n-t_m-2}u_m(z) \\
& + \alpha_{m-1,0}u_{m-1}(z) + \alpha_{m-1,1}zu_{m-1}(z) + \cdots + \alpha_{m-1,t_m-t_{m-1}-1}z^{t_m-t_{m-1}-1}u_{m-1}(z) + \cdots \\
& + \alpha_{1,0}u_1(z) + \alpha_{1,1}zu_1(z) + \cdots + \alpha_{1,t_2-t_1-1}z^{t_2-t_1-1}u_1(z) \\
& + \alpha_{0,0}u_0(z) + \alpha_{0,1}zu_0(z) + \cdots + \alpha_{0,t_1-t_0-1}z^{t_1-t_0-1}u_0(z).
\end{aligned} \tag{18}
$$

This implies that $z^{n-t_m-1}u_m(z) = \alpha_m(z)u_m(z) + \alpha_{m-1}(z)u_{m-1}(z) + \cdots + \alpha_0(z)u_0(z)$, where $\alpha_m(z) = \alpha_{m,0} + \alpha_{m,1}z + \cdots + \alpha_{m,n-t_m-2}z^{n-t_m-2}$ and $\alpha_i(z) = \alpha_{i,0} + \alpha_{i,1}z + \cdots + \alpha_{i,t_{i+1}-t_i-2}$ and $\deg(\alpha_i(z)) \le t_{i+1} - t_i - 1$ for all i, $0 \le i \le m-1$. Then, by multiplying equation (18) by $c^{\rceil - i_{m-1}}$, we get

$$z^{\rceil - t_m - 1}c^{\rceil - i_{m-1}}u_m(z) = \alpha_m(z)c^{\rceil - i_{m-1}}u_m(z). \tag{19}$$

Then, the degree of LHS of equation (19) is $n-1$ but that of RHS is at most $n-2$ which is a contradiction. Therefore, $z^{n-t_m-1}u_m(z)$ cannot be expressed as a linear combination of elements of S'. We can apply similar arguments to prove that none of $z^{t_m-t_{m-1}-1}u_{m-1}(z), z^{t_{m-1}-t_{m-2}-1}u_{m-2}(z), \ldots, z^{t_1-t_0-1}u_0(z)$ can be expressed as a linear combination of elements of S'. Therefore, we get that S' is linearly independent, and hence, it is a minimal spanning set for C. It follows that rank$(C) = n - t_0$.

The following theorem determines all the MDS cyclic codes having a length n over R.

Theorem 13. *A cyclic code C having a length n over R is an MDS if and only if it is principally generated by a monic polynomial and $Tor_0(C)$ is an MDS cyclic code having a length n over $\mathcal{T}$ with respect to Hamming metric.*

Proof. Let $C = \langle u_0(z), u_1(z), \ldots, u_m(z)\rangle$ be an MDS cyclic code having a length n over R such that $u_j(z)$, $0 \le j \le m$ are polynomials as in Theorem 10. Since C is an MDS, therefore, $|C| = |R|^{n-d_H(C)+1}$. By using Theorem 7, we have $p^{s(n)\rceil - ni_m - t_0k_0 - t_1k_1 - \cdots - t_mk_m)} = p^{s\rceil(n-d_H(C)+1)}$ which implies that $ni_m + t_0k_0 + t_1k_1 + \cdots + t_mk_m = \rceil(d_H(C) - 1)$. Thus, we can conclude that $t_j = 0$ for $1 \le j \le m$ and $i_m = 0$ because $i_m + k_0 + k_1 + \cdots + k_m = \rceil$ and $t_m > t_{m-1} > \cdots > t_0 \ge d_H(C) - 1$. This implies that C is principally generated by a monic polynomial and $t_0 = d_H(C) - 1$. By using Theorems 7 and 8, we have $|\mathcal{T}|^{(n-d_H(Tor_0(C))+1)} = p^{s(n-d_H(Tor_0(C))+1)} = p^{s(n-d_H(C)+1)} = p^{s(n-t_0)} = |Tor_0(C)|$. Thus, $Tor_0(C)$ is an

$|Tor_0(C)| = |\mathcal{T}|^{(n-d_H(Tor_0(C))+1)}$. By using Theorems 7 and 8, we can conclude that $|R|^{n-d_H(C)+1} = p^{s\rceil(n-d_H(Tor_0(C))+1)} = p^{s\rceil(n-t_0)} = |C|$, i.e., C is an MDS cyclic code over R.

The following lemma by Sharma and Sidana determinesthe Hamming distance of a cyclic code C of length $n'p^r$, MDS cyclic code over the residue field $\mathcal{T}$.

Conversely, suppose a cyclic code C having a length n over R is principally generated by a monic polynomial, say $u_0(z)$ as obtained in Theorem 10 and $Tor_0(C)$ is an MDS code over $\mathcal{T}$. Then, this means that i_0 � 0 and

n', p 1, and $r \geq 1$ over a finite chain ring R as given inreference [27]. □

$1,$ $if\ t_0$ � $0,$

Lemma 2 (see [27]). *Let C be a cyclic code having a length n � $n'p^r$ for (n', p) � 1 and $r \geq 1$ over R. Then, we have*

$$l + 2, \quad if\ lp^{r-1} + 1 \leq t_0 \leq (l+1)p^{r-1}, \text{ with } 0 \leq l \leq p-2,$$

$$(i+1)p^k, \quad if\ p^r - p^{r-k} + (i-1)p^{r-k-1} + 1 \leq t_0 \leq p^r - p^{r-k} + ip^{r-k-1}, \text{ with } 1 \leq i \leq p-1 \text{ and } 1 \leq k \leq r-1.$$

We use Lemma 14 mentioned above to determine all MHDR cyclic codes of length $n'p^r$, n', p 1 and $r \geq 1$ over R in Theorems 15 and 16.

Theorem 3. *A cyclic code C of length $n'p$, (n', p) � 1 over*

(i) for k � $r-1$, t_0 � $p^r - p + i$, $1 \leq i \leq p-1$, the Hamming distance of C is $(i+1)p^{r-1}$. C is an MHDR code if and only if $(i+1)p^{r-1}$ � n - rank$(C) + 1$ � $t_0 + 1$ by using Theorem 12. Then, we have $p - p + i$ � t_0 � $(i+1)p^{r-1} - 1$. It

a finite chain ring R is an MHDR code. (–) � (+)(–)

Proof. Let C be a cyclic code of length $n'p$, (n', p) � 1 over R.By Lemma 14, we have follows that p p^{r-1} 1 i 1 p^{r-1} 1 , which implies that i p 1, since $p^{r-1} \neq 1$. Then, C is an MHDR for t_0 p^r 1. It can be easily seen that forother values of t_0, C is not an MHDR code. □

Theorem 4. *Let C be an MDS cyclic code having an arbitrary length over R. Then, C is also an MHDR code over R.*

which implies that $d_H(C)$ � $t_0 + 1$ � $n -$ rank$(C) + 1$ for$0 \leq t_0 \leq p-1$ by using Theorem 12. Hence, a cyclic code of

Proof. Let C be an MDS cyclic code having an arbitrary length length $n'p$, (n', p) � 1 over R is always an MHDR code. □

Theorem 5. *Let C be a cyclic code having a length n � p^r, $r > 1$ over R. Then, C is an MHDR if and only if* $t_0 \in 0, 1, p^r - 1$.

Proof. By Lemma 14, we have the following:

(i) for t_0 �, the Hamming distance of C is 1, which isthe same as n rank C 1 by using Theorem 12. So, C is an MHDR code. – ( ) +

(ii) for $lp^{r-1} + 1 \leq t_0 \leq (l+1)p^{r-1}$ with $0 \leq l \leq p-2$, the Hamming distance of C is l 2. Here, C is an MHDR if and only if d_H C n rank C 1, i.e., l 1 t_0 by using Theorem 12. Then, lp^{r-1} $1 \leq t_0$ would imply lp^{r-1} $1 \leq l$ 1, i.e., l p^{r-1} 1 ≤ 0. It follows that l p^{r-1} 1 ( ) + 0, which implies l 0, since $p^{r-1} \neq 1$. Then, C is an MHDR if and only � t_0 � 1. +

n over R. By Theorem 13, C is principally generated by a monicpolynomial over R say $u_0(z)$ with degrees t_0 and i_0 � 0 and Tor $_0(C)$ is also an MDS code over $\mathcal{T}$. Then, we have �

$$Tor_0(C) \text{ � } p^{s(n - d_H(C)+1)}. \tag{22}$$

Also, from Theorem 7, we have

$$Tor_0(C) \text{ � } p^{s(n - t_0)}. \tag{23}$$

Equations (22) and (23) together with Theorem 12 imply that $d_H(C) = t_0 + 1 = n - \text{rank}(C) + 1$. Therefore, C is an MHDR cyclic code over R. □

However, Example 1 shows that the converse of the abovementioned statement is not true.

Example 1. Let $R = Z_5 + 5Z_5$. Let $C = \langle 5, (z-1)^{24}\rangle$ be a cyclic code having a length $n = 25$ over R. Here, $i_0 = 1$, $i_1 = 0$, $t_0 = 0$, $t_1 = 24$, $\text{rank}(C) = 25$, and $d_H(C) = 1$.

By using Theorem 16, we see that C is an MHDR cyclic code over R. However, C is not an MDS code, since it is not principally generated (using Theorem 17).

Example 2. Let $R = Z_5 + 5Z_5$. Let $C = \langle (z-1)^{24}\rangle$ be a cy- clic code having a length $n = 25$ over R. Here, $i_0 = 0$, $t_0 = 24$, $\text{rank}(C) = 1$, and $d_H(C) = 24$. By using Theorem 16, we see that C is an MHDR cyclic code over R. Also, C is an MDS code, since it is principally generated by a monic polynomial and $|Tor_0(C)| = 5 = |Z_5|^{n - d_H(Tor_0(C))+1}$ (using Theorem 13).

Example 3. Let $R = Z_2 + cZ_2 + c^2Z_2 + c^3Z_2$. Let $C = \langle (z^2-1) + c(z-1) + c^2(z-1) + c^3\rangle$ be a cyclic code having a length $n = 6$ over R. Here, $i_0 = 0$, $t_0 = 2$, $\text{rank}(C) = 4$, and $d_H(C) = 3$. It is principally generated by a monic poly- nomial and $|Tor_0(C)| = 2^4 = |Z_2|^{n - d_H(Tor_0(C))+1} = 2^{6-3+1} = 2^4$, so we see that C is an MDS code over R by using Theorem 13. Also, from Theorem 15, we see that C is also an MHDR code.

Example 4. Let $R = Z_2 + cZ_2 + c^2Z_2 + c^3Z_2$. Let $C = \langle c^2(z^3-1), c^3(z^2-1)\rangle$ be a cyclic code having a length $n = 6$ over R. Here, $i_0 = 2$, $t_0 = 3$, $\text{rank}(C) = 3$, and $d_H(C) = 2$. It is not generated by a monic polynomial, so by Theorem 13, C is not an MDS code. Also, from Theorem 15, we see that C is not an MHDR code.

Example 5. Let $R = Z_3 + cZ_3 + c^2Z_3$. Let $C = \langle c^2(z^2-1), c(z^2-1)^3 + c^2(z-1)\rangle$ be a cyclic code having a length $n = 18$ over R. Here, $i_0 = 2$, $i_1 = 1$, $t_0 = 2$, $t_1 = 6$, $\text{rank}(C) = 16$, and $d_H(C) = 2$. Since C is not generated by a monic polynomial, so by Theorem 13, it is not an MDS code. Also, from Theorem 16, we see that C is not an MHDR code.

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5. Conclusion

In this work, a unique set of generators for a cyclic code having an arbitrary length over a finite chain ring with an arbitrary nilpotency index has been established. The mini- mal spanning set and rank of the code have also been de- termined. Furthermore, sufficient as well as necessary conditions for a cyclic code having an arbitrary length to be an MDS code and for a cyclic code having a length which is not coprime to the characteristic of the residue field of the ring to be an MHDR code have been obtained. Some ex- amples of optimal cyclic codes have also been presented.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Disclosure

A preprint has previously been published [30].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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