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Research Article

Using a Definition to Find the First Derivative of Functions: Misconceptions and Remediation at the Collegiate Level

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Abstract

The aim of this research was to study and investigate first year university calculus students' misconceptions about how to use the definition of limit to find the first derivative of functions and then remediate these misconceptions by using APOS theory (Action, Process, Object, and Schema). Students (n = 82 female and male) were selected based on their performance in the first semester of university, i.e., purposive selection. Data obtained in the pretest indicated five types of misconceptions students had in handling the first derivative by finding the limit of function by the definition method. The researcher conducted a remedial plan on the concept of conjugates using APOS theory as an intervention for the remedy and recovery. The result of the post-test showed improvement and progress in the students' achievement, so the intervention was effective by using APOS Theory. The researcher recommended using APOS for similar situations

Keywords

Misconceptions in mathematics, APOS theory; functions; intervention; calculus I; derivative by limit (definition); conjugate

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Introduction

This article aims to clarify students' misunderstanding of using the definition technique to find the derivative informal process, i.e., to calculate it through the limit approach. The derivative of the function $f(x)$ concerning the variable x is the function whose value at x is $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, provided the limit exists (Thomas, S, 2010).

It is well known that the geometrical explanation and representation of this first derivative of *f*(x) at *x* is the limit of the slope line of the secant line from *x* to *x*+*h* as *h* approaches nearer and nearer to 0.

Defining a new strategy that will remedy the misconceptions that students have, and will also encourage thinking. The article will contribute to resolving a dominant disagreement between educationists of conceptual variation concerning the construction and consistency of students' mathematical information. It will help students understand the concept of finding derivatives of functions by using the definition, regardless of the root power. The study will contribute to this important and fundamental concept, allowing more detailed queries to be tackled about the nature of students' familiarity with misconceptions. It will also explain the part of methodology in remedial studies. Finally, the solution to enhancing students' understanding of this concept is by looking at how students from different universities employ it. It will enhance the improvement of the curricula, which in turn will be met with enthusiasm by colleagues, students, and potentially educational institutions along the way.

The first year of college is an important period in students' mathematical grasp, preliminary to Calculus-I. During this period, students likewise get confused due to unobserved misconceptions. The objective of this research is to specify students' misconceptions on limits while using them to find the derivative by definition, the reasons behind these misconceptions, and intervention to remedy these misconceptions by the APOS Theory (Action, Process, Object, and Schema) approach.

Literature Review

Calculus considered one of the most important subjects in schools, and universities, (Bressoud, D. M, 2021) because it has the basic concepts and facts that considered the prerequisites for the whole courses in the university, meanwhile students, and novice teachers still have many misconceptions in calculus such as the definition of the rate of change (Frank, K., & Thompson, P. W, 2021), limits (Bansilal, S., & Mkhwanazi, T. W, 2021), continuity (Perfekt, K. M, 2021) , differentiations (Bangaru, S. P., Michel, J., Mu, K., Bernstein, G., Li, T. M., & Ragan-Kelley, J,2021) , and integrations (Fernandez, A., & Mohammed, P,2021). Many researchers in math-education try to explore the reasons behind these misconceptions. For example, many students have no clear idea about the concept of function and its elements such as domain and range (McDowell, Y. L.,2021). Furthermore, Syarifuddin, A., and Sari, A. F. (2021) discovered mistakes teachers made when graphing functions. Avgerinos, E., & Remoundou, D. (2021) investigated the misconception of the rate of change and found students have difficulties in the language of mathematics. In the same event, both teachers and students have the same misconception when it comes to trigonometric functions, derivatives, and intergradations (Toh, T. L., Tay, E. G., & Tong, C. L., 2021). So, to tackle these misconceptions in calculus in an effective manner, many researchers use APOS Theory (Action, Process, Object, and Schema). For example, Sallah, E. K., Sogli, J. K., Owusu, A., & Edekor, L. K. (2020) used it to fix students' misconceptions in integrals, while Kwadzo, S. E., Sogli, J. K., & Owusu, A. (2020) used it to fix students' misconceptions in differential, and zkaya, M., & Isleyen, T. (2021) used it to fix students' misconceptions about the function concept, and the current study will use it to remedy students' misconceptions about finding the first derivative by using the definition, i.e., limit notations.

Background of the Research

The limit concept is vital to accepting pre-calculus and calculus. Tall (1992) confirmed, "The idea of limit signifies a progression to a higher level of mathematical thinking." Other concepts like continuity, rate of change, derivatives, and integration are the applications of the limit concept. Many studies, such as Davis and Vinner (1986), Tall, (1992), Jordan, (2005), zkan, E. M., & Ünal, H. (2009), have shown that students have a serious misconception about the limit and its applications. Moreover, many studies, such as Uygur & zdas (2005) and Orton (1983), showed that the derivative is very difficult for students, and gets worse with advanced concepts such as composites of function derivatives (Tall, 1993) and chain rule (Gordon, 2005; Uygur & Zdas, 2007).

The Problem Statement

It is simple to diffuse the problem (or mathematical errors), but it is more difficult to completely solve it using traditional methods, because students' mathematical beliefs have a long time on their schematic thinking (Nussbaum, E. M.,2021). So, what to do with this student's mentality? The answer took many times, efforts, and years, but finally, researchers in math education, such as Borji, V., and Martnez-Planell, R. (2020), used APOS theory. There are many justifications for using APOS theory for dealing with misconceptions; it targets the false mathematical facts and conceptions (Herawaty, D., Widada, W., Handayani, S., Febrianti, R., & Anggoro, A. F., 2020), it creates cognitive conflict in the minds of students (Widada, W., Herawaty, D., Nugroho, K. U. Z., & Anggoro, A. F. D., 2019), and it lets students follow certain logical thinking methods (Afgani, M. W., Suryadi, D., & Dahlan, J. A., 2017). Many studies used APOS to improve students' abilities in mathematics. For example, (Arnawa, I. M. et al., 2007) used this theory to improve students' ability in algebra; others used it in calculus such as (Borji, V., et al.2018, Martínez-Planell, R., 2015, Tokgoz, E.2015, Tokgöz, E. 2016, Borji, V., et al. (2016) Zhu, W., et al. (2017).
But none of them used it to correct students' misconceptions about finding the first derivative using the limit definition, i.e., no such study addresses students' misconceptions about conjugate concepts. The following main problem is linked to finding the first derivative by definition of limit, so misconceptions need to be examined to analyze the actual misconception: what causes these misconceptions? What is the impact of the APOS method of treatment for misconceptions on students? So our research hypothesis is concerned about students who have misconceptions about finding the first derivative by limit, if there is a significant statistical difference between post-test and pre-test results of students in outlining the misconceptions of finding the first derivative by limit, and if there is an impact of the proposed APOS method. This research aims to detect misconceptions and the effectiveness of the program proposal to teach limit by the APOS approach, and measure the impact of treatment misconceptions on finding the first derivative by the limit that has been done by students. To detect the cause of misconceptions, propose ways to fix misconceptions, measure the effectiveness of the proposed program in teaching students, and measure the impact of the proposed program on treating misconceptions to students. To achieve these aims, the following points are expressed:

1. To collect data on the students' misunderstanding of the limit, utilizing the first exam on Calculus I
2. Interview
3. Analyze the gathered data to determine the misconceptions that students have
4. Apply the intervention to remedy these misconceptions.
5. Measure the effects of the intervention

The Research Procedure

This research was run at the university site. The researcher is a lecturer in calculus at this establishment. Students are majoring in a BS in Engineering. The main topics of Calculus Ɨ are: Limits, Limits involving infinity, Parametric equations, Derivatives, Chain Rules, Continuity, Implicit Differentiation, Rates of Change, Related Rates, Tangent Lines, Extreme values, Mean value theorem, graph shape, optimization, indefinite integrals, definite integrals, definite integrals, definite integrals, definite integrals, definite integrals, definite integrals, definite integrals, definite integrals, Integration by substitution, Fundamental theorem area, The textbook used in the class is Thomas' Calculus Eleventh Edition (Thomas, S., 2010).
Students' teaching is through the English medium. For the study, first-year students were used. The research steps can be shown in the following figure 1:

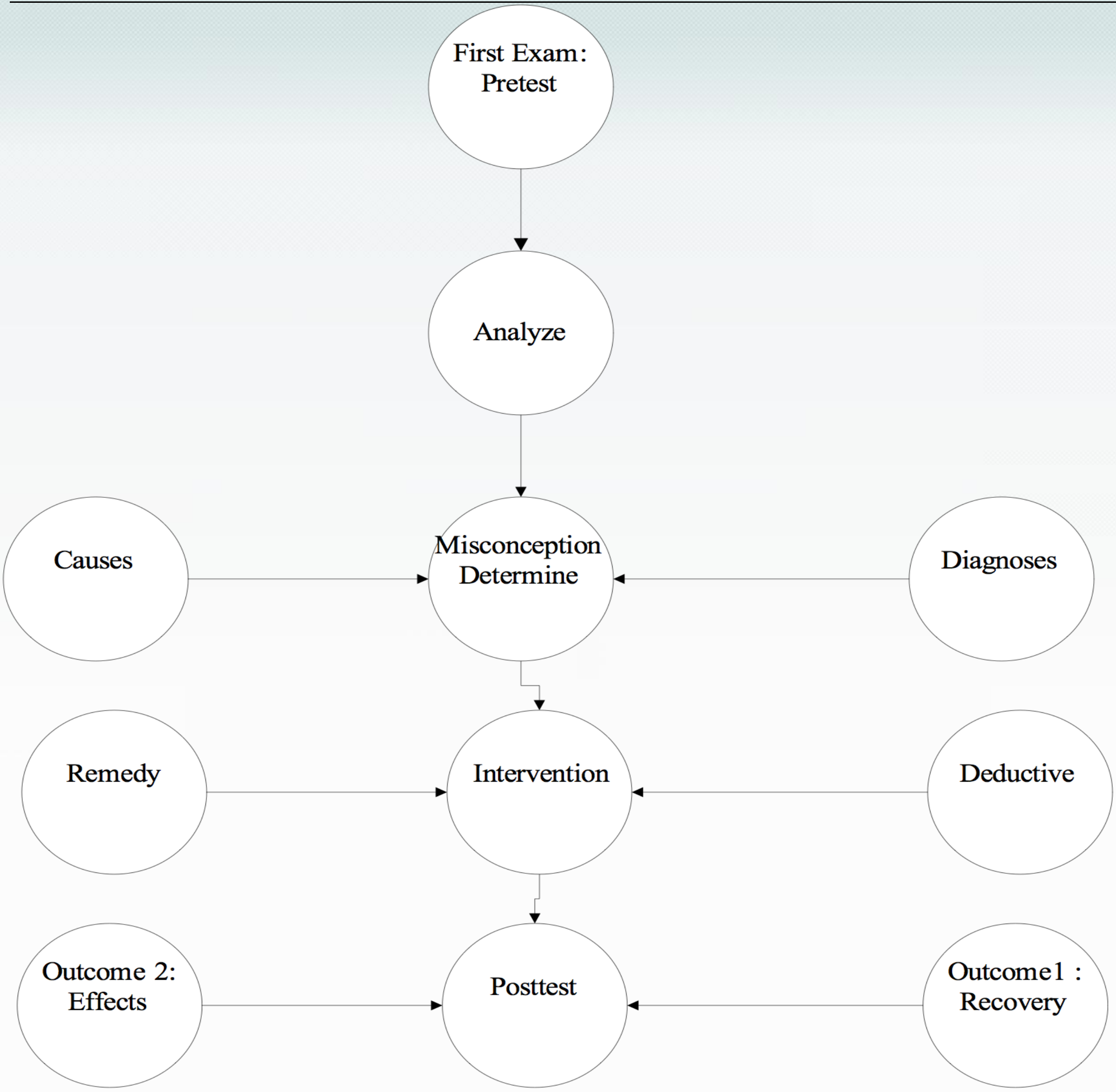

Figure 1. The research steps

Figure 1, shows the whole steps that have been taken in this research, from applying the pretest to diagnose students' weaknesses and strengths in finding the first derivative by the limit approach, then analyzing the misconceptions to the reasons behind them, then making intervention by APOS theory, and finally applying the posttest for the purpose of checking the recovery from misconception and the effectiveness of intervention.

Research Importance

The importance of this research comes from the hopes of improvement. Through follow-up of students to understand the limit and its applications, it also helps students to progress well by using the limit to find the first derivative, assists teachers in professional growth and development, and raises scientific understanding and attention to the relevant educational process through the development of proposals and solutions.

Theoretical Framework

Our framework entails three components (Asiala et al., 1996), namely: theoretical analysis, design, implementation of instructions, and collection and analysis of data.

Theoretical Analysis

This plan is based on the theoretical framework of the APOS theory (Weller et al., 2003; Dubinsky, 2010). APOS theory is used in many ways to describe the learning of concepts, curriculum design, evaluation, and so on. The researcher provided appropriate examples of conjugate concepts to simplify APOS.

The basic element of APOS is an action, which is the first step in grasping mathematical concepts from the external environment, i.e., stimuli received by the senses and reacted to by the learner. For example, a learner who wants to reflect on the conjugate of numbers such as $3-2$, can do slightly extra than complete the action, $3^2-2^2=(3-2)(3+2)$, is measured to ensure action acceptance of the conjugate concept and understanding that the conjugate of $3-2$ is$3+2$ and Vic versa, because multiplication of two parts gives us the new formula that represents differences between two perfect squares. In the same action, a leaner can perform rational power of conjugate, such as $3-2=(3^{1/3}-2^{1/3})(32^{/3}+(3.2)^{1/3}+2^{2/3})$.

The second part of APOS is the process, so if the learner learned about the concept from the external environment, she/he will reflect and repeat this action many times to interiorize this concept on her/his mental memory. For example, a learner who reflects and repeats actions in the conjugate of numbers will perform more formulas such as$u^2-v^2=(u-v)(u+v)$, and $u-v=(u^{1/3}-v^{1/3})(u^{2/3}+(uv)^{1/3}+u^{2/3})$.

The third part of APOS is the objective part, which is about the awareness of acquisition of the process, which includes action. In this step, the leaner transforms the concept explicitly, or implicitly, by encapsulating the process into the mental object. For example, a student can easily and mentally handle any type of conjugate, such as$u^n-v^n=(u-v)(u^{n-1}+u^{n-2}v+\cdots+u^2v^{n-3}+uv^{n-2}+v^{n-1})$, or $u^{\frac{n}{n}}-v^{\frac{n}{n}}=\left(u^{\frac{1}{n}}-v^{\frac{1}{n}}\right)\left(u^{\frac{n-1}{n}}+u^{\frac{n-2}{n}}v^{\frac{1}{n}}+\cdots+u^{\frac{2}{n}}v^{\frac{n-3}{n}}+u^{\frac{1}{n}}v^{\frac{n-2}{n}}+v^{\frac{n-1}{n}}\right)$

The fourth part is the schema. It is the holistic construction of mathematical concepts, including actions, processes, and objects, that is necessary to be structured and related to a clear understanding of mathematical concepts. So the schema is considered logical and meaningful in understanding for the learner. It provides the student with a method of leading key; when accessed by an exact mathematical condition, any misleading information lets the schema collapse, and no logical thinking accrues. For example, when a student is given a mathematical task such as finding the derivative by definition of the function$f(x)=\sqrt[3]{1-x}$, Students must know all the steps to solve the problem. So the student applies for The derivative is obtained by substituting, multiplying by the right conjugate, canceling the common factor from both the numerator and the dominator. To know more about APOS theory, we need to make a genetic decomposition of the conjugate concept.

Method

Data Collection

Data was collected in the following steps:

First of all, a pretest written test was conducted to diagnose the students' (n = 82) misconceptions about the concept of derivative by using the definition of limit. The result of this test was analyzed to determine these misconceptions. Five types of misconceptions were detected as shown in table (1).

Secondly, a semi-interview was conducted by asking students why they chose this misconception in finding the limit of function. Then it was classified as 1) multiply the numerator and dominator by the conjugate as a square for any type of function (n = 75%), 2) Signs (minus and): students make a mistake in adding or subtracting (6%), 3) The Quotient Rule: students cancel numerator and dominator although there are no common factors between them (2%), 4) Direct substitution of "h", and getting an undefined value (1%), and 5) No solutions (1%).

Finally, a posttest written test was conducted and analyzed to compare its results with the pretest. Participants were first-year engineering students (n = 82, mixed female and male) at the University of Polytechnic, chosen by a purposive method. The average age of these students was 18 years old. They came from several schools. These students learned the concepts of limit and derivative in 12th grade, so it was assumed these students have the basic concepts of finding the first derivative of a function by the definition. Furthermore, these students have examined these concepts twice in 12th grade. The first testing was informal (i.e., a pilot test conducted by their local school, and the second testing was formally conducted by the Ministry of Education, called the General Exam Test. The result of GET differentiated students in which field they would continue their studies, so if a student got 90% or above in GET, they would join the college of engineering. For the motivation of this feedback, students were questioned to find the derivative by definition of the function, i.e., $f(x) = \sqrt[3]{1-x}$ (i.e. $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. The data table was built based on the following categories: solutions that carry misconceptions and blank paper. The researcher examined and analyzed the data. Moreover, the first exam that has this problem was designed by 7colleagues, who are experts in this field.

From the statistical analysis, five main types of misconceptions have been recognized. The first type occurred when students multiplied the numerator and dominator by the conjugate in square form rather than the conjugate in cubic form. The second category was that students misused signs (negative vs. positive). The third category was using the quotient rule, which is not allowed at this stage. The fourth category was direct substitutions. The fifth and final category was "no solutions." Table 1 summarizes students' misconception strategies in the process of finding the derivative by definition of the function:$f(x) = \sqrt[3]{1-x}$ (i.e. $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$):

Table 1

Students' misconceptions

Strategies Names	Example	Percentage
1) Multiply the numerator and dominator by the conjugate as a square for any type of function.	$\sqrt[3]{1-x-h} + \sqrt[3]{1-x}$	72%
2) Signs (minus and): the student makes an addition or subtraction error.	$\sqrt[3]{1-x+h} \pm \sqrt[3]{1-x}$	6%
3) The Quotient Rule: students cancel numerator and dominator although there are no common factors between them.	The Quotient Rule	2%
4) Direct substitution of "h" and getting an undefined value.	$\sqrt[3]{1-h-h} + \sqrt[3]{1-h}$	1%
5) No solutions		1%
6) Correct answer		18%

Table 1 shows that the majority of students have misconceptions (82%), when multiplying the numerator and dominator by the conjugate as a square for any type of function. They do not know that the cubic root conjugate is different than the conjugate as a square. On the other hand, only 18% of students understand the concept.

The Reasons Behind These Misconceptions Are

The researcher conducted interviews with 10 randomly selected students. To learn the reasons for these misconceptions, select 4 from the lower performance scale, 4 from the middle performance scale, and 3 from the highest performance scale. The tool of this interview was the pretest exam. The researcher asked each student, according to her or his written answer, "Why did you write down this answer?". The interview of each student took about one minute; it was videotaped, then transcribed, and then categorized according to the common idea of the students' thinking. The majority of students mentioned the following notes about the reasons for these misconceptions:

One student (10%) believes that the conjugate is the same for any root.
2) Two students (20%) claim that their teachers in schools ignore this type of function, such as cubicles.
3) Three students (30%) admit to memorizing mathematics without understanding it.
4) Three students (30%) confess that they don't care about mathematics concepts such as conjugates.
5) One student (10%) believes that the expansion of cubic root brackets was more difficult to handle.

Iinterventions

Many studies have stated the effective interventions of creative methods such as the APOS method and how they promote students' learning of many difficult mathematical concepts effectively. Becker (1992) and Jennifer (2000), for example, used APOS theory intervention to help students overcome misconceptions about limits.
APOS was tested in many mathematics classrooms and gave fruitful results (Dubinsky and McDonald, 2001), so the researcher was highly motivated to use APOS to enhance his students' performance and understanding of the most important concept in calculus, which is limit and derivatives.

Designing and Implementation of Instructions

Designing and implementation of instruction depend on the genetic decomposition of conjugate concepts, so the genetic decomposition of a conjugate concept is an organized regular ~~of~~ mental ideas that influence and define how the concept can develop in the mind of a learner (Asiala et al., 1996). So, a genetic decomposition assumes the precise actions, processes, objects, and schema that play a part in the building of a conceptual mathematical state of a learner's mentality. The genetic decomposition activities for the conjugate concept are shown below.
The researcher conducted one lecture using the APOS approach as follows:
The lecturer divided the students into groups of five and asked each group to work together to complete a worksheet that required them to compose and decompose quantities in order to find the conjugate of each formula type based on its power (exponential).Table 1 below shows the worksheet contents.
Action: The conjugate concept can begin with an integer power of numbers followed by rational power, so it is natural to follow these deductive activities to make sure our learner grasps it simply and naturally. Table 2, shows a proposed decomposition that may be used in instruction by dividing students into groups and implementing it.

Table 2
Conjugate concept by using numbers formula.

Number types		Conjugate parts	Check it!
Integer power	Rational power		
$3^2-2^2=(3-2)(3+2)$	$3-2=(3^{1/3}-2^{1/3})(3^{2/3}+(3.2)^{1/3}+2^{2/3})$.	$(3^{1/3}-2^{1/3}),(3^{2/3}+(3.2)^{1/3}+2^{2/3})$	*3^2-2^2 is equal to $(3-2)(3+2)$?*

Table 2 represents a sample of some examples that have been used during the intervention of this research.

Process: came after an action, so a student can generalize numbers to a more sophisticated algebraic formula.

Table 3
shows such activities that achieve process.

Formula types Integer power	Conjugate parts	Check it!
$u^2 - v^2 = (u-v)(u+v)$	………	………
$u^3 - v^3 = (u-v)(u^2+uv+v^2)$	………	………
$u^4 - v^4 = (u-v)(u^3+u^2v+uv^2+v^3)$	………	………
$u^5 - v^5 = (u-v)(u^4+u^3v+u^2v^2+uv^3+v^4)$	………	………
$u^n - v^n = (u-v)(u^{n-1}+u^{n-2}v+\cdots+u^2v^{n-3}+uv^{n-2}+v^{n-1})$	………	………

Table 3 represents the algebraic formula that has been discovered by students in Table 2. They used the induction approach to make this discovery with the teacher's help, and the same activities are designed for rational power. So, students can generalize each formula to *n* power.
Objective: furthermore, the student can generalize the conjugate theory to any formula with any degree and for any terms of power, so it is expected to construct this mental image in their mind: $u^n - v^n = (u-v)(u^{n-1}+u^{n-2}v+\cdots+u^2v^{n-3}+uv^{n-2}+v^{n-1})$…….(1)
With a slight modification to formula (1), we can write the following: $u^{\frac{n}{n}} - v^{\frac{n}{n}} = \left(u^{\frac{1}{n}} - v^{\frac{1}{n}}\right)\left(u^{\frac{n-1}{n}} + u^{\frac{n-2}{n}}v^{\frac{1}{n}} + \cdots + u^{\frac{2}{n}}v^{\frac{n-3}{n}} + u^{\frac{1}{n}}v^{\frac{n-2}{n}} + v^{\frac{n-1}{n}}\right)$….(2)

1) By using formula (2), we can solve all the conjugates of all power roots. For example, $u^{\frac{2}{2}} - v^{\frac{2}{2}} = \left(u^{\frac{1}{2}} - v^{\frac{1}{2}}\right)\left(u^{\frac{1}{2}} + v^{\frac{1}{2}}\right)$.

2) $u^{\frac{3}{3}} - v^{\frac{3}{3}} = \left(u^{\frac{1}{3}} - v^{\frac{1}{3}}\right)\left(u^{\frac{2}{3}} + (uv)^{\frac{1}{3}} + v^{\frac{2}{3}}\right)$, and so on…

None example (misconception):
$u^{\frac{3}{3}} - v^{\frac{3}{3}} \neq \left(u^{\frac{1}{3}} - v^{\frac{1}{3}}\right)\left(u^{\frac{1}{3}} + v^{\frac{1}{3}}\right)$, why?

Schema This is the highest level of APOS, so it is expected of students to tackle and solve any derivative by limit definition, using $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, substituting function on it, factorizing, multiplying by right conjugate, canceling the common factor from both the numerator and the dominator, and getting the derivative of a function. At this stage, the researcher encourages students to check if the answers are correct or not by using GeoGebra software on their Smartphone or by using the general rules of the derivative.

Students Take Home Assignments

The road map of this assignment method is complementary to the previous activities, that aim to empower students in general formula, examples, non-examples, applications. This method aims to create cognitive conflict between the misconceptions that students did in the pretest exam and to learn and reflect on the true concept.
Table 4 represents the student group discussion. It was about examples, none-examples, and $(u-v)$ what is called the conjugate of $(u+v)$, and vice versa, the same thing for the cube conjugate, *i.e.*, $(u-v)(u^2+uv+v^2)$.
Now we can generalize to n power as follows. This formula is well known
$u^n - v^n = (u-v)(u^{n-1}+u^{n-2}v+\cdots+u^2v^{n-3}+uv^{n-2}+v^{n-1})$…….(1)
With slight modification on formula (1), we can write the following:
$u^{\frac{n}{n}} - v^{\frac{n}{n}} = \left(u^{\frac{1}{n}} - v^{\frac{1}{n}}\right)\left(u^{\frac{n-1}{n}} + u^{\frac{n-2}{n}}v^{\frac{1}{n}} + \cdots + u^{\frac{2}{n}}v^{\frac{n-3}{n}} + u^{\frac{1}{n}}v^{\frac{n-2}{n}} + v^{\frac{n-1}{n}}\right)$….(2), which is easy to prove by the APOS method.

Table 4
Students home worksheet assignments.

Formula type	Example	None-Example
$u^2 - v^2 = (u-v)(u+v)$	………	……….
$u^3 - v^3 = (u-v)(u^2+uv+v^2)$	……….	………..
$u^4 - v^4 = (u-v)(u^3+u^2v+uv^2+v^3)$	……….	…….
$u^5 - v^5 = (u-v)(u^4+u^3v+u^2v^2+uv^3+v^4)$	……..	…….
$u^n - v^n = (u-v)(u^{n-1}+u^{n-2}v+\cdots+u^2v^{n-3}+uv^{n-2}+v^{n-1})$	……..	………

By using formula (2), we can solve all the conjugate of all power roots, for examples:

1) $u^{\frac{2}{2}} - v^{\frac{2}{2}} = \left(u^{\frac{1}{2}} - v^{\frac{1}{2}}\right)\left(u^{\frac{1}{2}} + v^{\frac{1}{2}}\right)$.
2) $u^{\frac{3}{3}} - v^{\frac{3}{3}} = \left(u^{\frac{1}{3}} - v^{\frac{1}{3}}\right)\left(u^{\frac{2}{3}} + (uv)^{\frac{1}{3}} + v^{\frac{2}{3}}\right)$, *and so on…*
None example :
$u^{\frac{3}{3}} - v^{\frac{3}{3}} \neq \left(u^{\frac{1}{3}} - v^{\frac{1}{3}}\right)\left(u^{\frac{1}{3}} + v^{\frac{1}{3}}\right)$, *why?*

Application

We will use the above approach to find the first derivatives by definition as on the following example:
Example: Find the first derivative by using the definition for the function

$$f(x) = \sqrt[3]{1-x}$$

Before we solve note that $f(x+h) = \sqrt[3]{1-(x+h)}$, *so we have the case:*
So, in example (2) let $u = (1-(x+h)), and v = (1-x)$, *then we have*

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt[3]{1-(x+h)}-\sqrt[3]{1-x}}{h} = \frac{0}{0}$$

$$= \lim_{h\to 0}\frac{\sqrt[3]{1-(x+h)}-\sqrt[3]{1-x}}{h}\,\frac{\left(\sqrt[3]{1-(x+h)}\right)^2+\sqrt[3]{1-(x+h)}\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2}{\left(\sqrt[3]{1-(x+h)}\right)^2+\sqrt[3]{1-(x+h)}\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2}$$

$$= \lim_{h\to 0}\frac{1-(x+h)-(1-x)}{h\left(\left(\sqrt[3]{1-(x+h)}\right)^2+\sqrt[3]{1-(x+h)}\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2\right)}$$

$$= \lim_{h\to 0}\frac{-h}{h\left(\left(\sqrt[3]{1-(x+h)}\right)^2+\sqrt[3]{1-(x+h)}\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2\right)}$$

$$= \lim_{h\to 0}\frac{-1}{\left(\sqrt[3]{1-(x+h)}\right)^2+\sqrt[3]{1-(x+h)}\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2} = \frac{-1}{\left(\sqrt[3]{1-(x+0)}\right)^2+\sqrt[3]{1-(x+0)}\sqrt[3]{1-x}+\left(\sqrt[3]{1-x}\right)^2}$$

$$= \frac{-1}{3\left(\sqrt[3]{1-x}\right)^2}$$

Home Work: Find the first derivative by using the definition for the function

$f(x) = \sqrt[4]{1-2x}$, $g(x) = \sqrt[5]{x^3+3x}$

Results

After conducting one lecture using the deductive approach, the researcher applied a post-test to students to gauge the effects of his intervention. The post-test has two different problems. The first problem for section (A) was to find the first derivative by using the definition for the function$f(x) = \sqrt[4]{x-1}$, and the second problem for section (B) was to find the first derivative by using the definition for the function$(x) = \sqrt[3]{1 - sin\,(x)}$. (*Hint: After multiplying by conjugate, use this identity: [sin (x + h) - sin x] / h = 2 cos[(2 x + h)/2] sin (h/2) / h)*
The result indicates there is a significant statistical difference between post-test and pre-test by applying a paired test as in the following table 5:

Table 5
Paired t- test between pre-test & post-test.

Test type	Mean	Std. Deviation	N	t	Df	Sig.
Pretest	0.25	0.43	82	8.25	81	0.000
Posttest	0.67	0.35	82			

Table5 shows progress in students' achievement since the mean of the pretest was 25%, while the mean of the posttest was 67%, which means there was a change in students' knowledge and understanding of finding any first derivative by definition or limit concept. Furthermore, the standard deviation was reduced from 43% to 35%, i.e., reducing the variance between students in finding the first derivative by definition.
Applying paired t-test between pretest and posttest shows a clearly significant difference due to the mean of the posttest as in table 5

Table 6
Correlation between Pretest & Posttest

	N	Correlation	Sig.
Pretest & Posttest	82	0.34	0.002

Table 6 shows the same result obtained from the correlation between pretest and posttest was r = 0.34, which is much weaker and significant due to the posttest.

Discussion

The results demonstrated the importance of using APOS theory to correct students' misconceptions of mathematics concepts such as finding the derivative of functions using the defining of limit i.e., $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$, So decomposition the procedure of finding the limit to action, process, object, and schema can help teachers determine the wrong step that students made. Then we can save our time and efforts in dealing with students' misconceptions. Thus, many studies used APOS, such as Moon, K. (2020), Kamid, K., Huda, N., Rohati, R., Sufri, S., Iriani, D., & Anwar, K. (2021). Kamid, K., Huda, N., Rohati, R., Sufri, S., Iriani, D., & Anwar, K. (2021), Panjaitan, B. (2020), and Sampanis,. (2020).
Moreover, this study demonstrates a significant correlation between pretest and posttest, i.e., a strong alignment between the parts of studies and indicates smooth transitions between the actions that have been taken by the researcher.
The analysis of this study supports the APOS theory, so we have a significant difference between the means of the pretest and posttest.
The data suggests improving in the performance of the students in calculus, and this encourage the researcher to us APOS theory in his teaching method as an innovative approach, in conclusion the research was assumed to find responses to the following inquiries:

1) What misconceptions did the student have about finding the first derivative by limit? 2) What causes these misconceptions?
3) What impact did the deductive method of treatment of misconceptions have on students?
This inquiry can now be replied to as follows:

Students have misconceptions about finding the first derivative by limit. There is a significant statistical difference between the posttest and pretest results of students in outlining the misconceptions about finding the first derivative by limit. There is an impact of the APOS methods on reducing these misconceptions to a minimal value.

Conclusion

Using a definition to find the first derivative of functions is not an easy task for students at university level, because they have many misconceptions due to many causes.
While these misconceptions are very difficult to detect and fix, the APOS theory is very efficient at doing this because it follows the logical human mind's thinking.
The most important evidence for this theory is to make a genetic decomposition of the concept of conjugate, then apply this decomposition in the classroom instructions.
According to the nice results, we have the researcher recommending:

To examine the role of the APOS method in further mathematics misconceptions,
To develop an instructional strategy by APOS that will reduce students' misconceptions
To conduct a diagnostic test on calculus for all first-year students to explore their misconceptions and remedy them.

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