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Research Article

Using Fractional Factorial Experiments 2(5-1) in Complete Randomized Block Design to Study the Factors Affecting the Acidity of Yoghurt

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Abstract

The study of design and analysis of experiments is one of the important studies that depend on experimentation through observation, data collection and analysis in order to reach high-accuracy scientific decisions and the lowest possible cost, and experiments are divided on the basis of the studied factors into Simple Experimental Design and factorial experiments Factorial Experimental Design. It is known that simple experiments are of limited use because they study the effect of one factor, and in most cases the researcher needs to study the effect of more than one factor at the same time, so he turns to factorial experiments, which allow estimating and testing each factor separately, in addition to measuring the effects of the interaction between factors. Although the factorial experiment provides large quantities of treatments, providing experimental units with a close degree of homogeneity represents a great difficulty for the researcher, especially when the number of factors is more than (4). Therefore, part of the treatments is taken according to the fractional factor experiment technique. The effect of (5) two-level factors (high and low) was studied in the fractional factorial experiment 2(5-1) (half-repetition) in the Complete Randomized Blocks. Data taken from Dairy Factory / Dairy Department. Skimmed milk powder (A), inoculation temperature (B), incubation temperature (C), incubation time (D) and fat percentage (E)). A half-repetition experiment was designed that includes main factors and some higher order interactions. Small and non-significant interactions were combined with the important main effects to obtain an estimate of the main effects and important interactions. The results showed by finding the analysis of variance table that the main effects (A, D) were significant in addition to the binary interaction (AD). The value of (P-Value) was less than the level of significance (0.05).

Keywords

Design and analysis of experiments, factorial half-repetition experiment, main effects and interactions with higher degrees, analysis of variance table, acidity of yoghurt

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Introduction

Experimental design, it is a series of steps that are saturated with the aim of collecting data or information and analyzing it statistically and arriving at conclusions that can be used. The science of designing and analyzing experiments is one of the branches of statistics and is concerned with the application of statistical methods in high-level experiments. It is concerned with planning and employing the available capabilities to put the appropriateness of experimental designs on a sound scientific basis that guarantees the possession of scientific decisions with a high degree of accuracy, progressing experiments on the basis of the studied factors to simple experiments and high experiments, and the experiments vary, transforming the way they work into a teacher experiment (in the laboratory or clinic) and a field experiment (field), and it varies according to the groups of samples on which the study is conducted. It is also possible to talk about experiments that require a long or short time to achieve the stability of their objective. In this chapter, we will address the basic concepts of global commercial design, or the mechanisms of planning and implementing experiments. And the requirements of good experimentation and methods of statistical analysis of experimental data.

2 -The Problem of the Study

1. The problem lies in the case of an increase in the number of treatments as the number of factors and their levels increase with the full factorial design, as it is sometimes difficult to provide homogeneous experimental material for a large number of experimental units.
2. There are researches that require in-depth knowledge of the levels of factors, because limiting the study to the factor without its levels hides many of its effects if not completely obscures them.

3 -The Aim of the Research

The importance of this study extends to the possibility of estimating the effects of the main factors and the important interactions of the lowest order of factorial experiments in the least time and lowest cost, using the fractional replications technique.

4 -Previous Studies

In (Bose & Nair, 1939) studied topics of analysis of variance, and analyzed the variance of the completed experiment in the process of designing the balanced incomplete Block design the factorial experiments using the ANOVA table (a suggested table), and stated that the balanced incomplete Block design It plays an important and significant role in the factorial experiment.
In the each of (Al-Hamdani and others) implemented a factorial experiment according to the design of randomized complete sectors (CRBD) and iterations of repetitions in the (sectors) this experiment included two main factors representing the first factor Nitrogen fertilizer and levels (0, 40, 80, 120) kg/dunum, representing the second factor with planting distances between traffic (40, 80, 120) cm. On the urban characteristics of the eggplant plant, while there were no significant differences between the cultivation distances used in the experiment on the urban characteristics of the same, and the results also showed increasing the productivity of the eggplant crop and a noticeable improvement in the yield characteristics when the plant is planted at a distance of 120 cm and fertilized with an amount of 120 kg/dunum.
In (Kuhfeld, Tobias, & Garratt, 1994) presented a paper dealing with the study of incomplete factorial experiments according to my complete randomized design (CRD) and randomized complete block design (CRBD). Exclusion of some treatment compounds used and it was concluded that there was a noticeable increase in the variance that was caused by the exclusion of some treatment compounds.
In (Kuhfeld et al., 1994) presented several suggestions that contribute to facilitating the design and analysis of experiments, especially experiments that require many calculations for sums of squares (SS), including factorial experiments (2^n) with total integration, by adopting Easy formulas and methods are assumed for how to fix the sign of the coefficients of the processors as well as to calculate the sums of squares of all complexes when the sum of the squares of error (S.S) is divided in the factorial experiment.
(Wilk & Kempthorne, 1956) studied the factorial experiment carried out according to the

randomized complete block design (CRBD). The experiment included two factors and two strings on the studied adhesion of the eggplant (Solanum Melongena L.). The results of the analysis showed that There is a significant difference between the concentrations of potassium mimed.
Also, researcher (Aldebs) in 2017, studied the most important factors affecting the increase in the productivity of eggplant under greenhouse conditions in Karbala governorate by designing randomized complete Block (CRBD) for a factorial experiment of type (3x5) that included two experimental factors, the first represented by eggplant strains. And it contained three levels, while the second represented by fertilization and contained eight levels, the experiment was applied in four Block, and the results of the study showed that there were significant differences in the factors of fertilization and strains, as well as that the sectors increased the amount of eggplant production, while no significant differences were diagnosed in the interaction of workers, as the researcher used orthogonal comparisons in Determining illiterate play, concentrations, factors affecting the guarded experience.

5- Factorial Experiments

Factorial experiment is not a design, but merely the organization of transactions within any of the noble designs, and through the factorial experiment, the opportunity to study and Impact evaluation common to the two or more factors holding their participation together in the same experiment on the basis that it is an opportunity to evaluate and study the effect of the interaction between the factors under study in the global experiment. The effect resulting from the participation of the variables together is completely different and the light effect is greater than that of the experiment variables if each variable is taken individually (Wilk & Kempthorne, 1956; Wold, 1978)

5.1 Advantages of Factorial Experiments [7][4]

Advantages of high experiments when compared with simple experiments are:

- It has a high expiation because each observation provides us with information about all the factors that are lawful in experience.
- It is easy to analyze as there is only one experimental error.
- Increasing confidence in the experiment compared to simple experiments as a result of the low value of the experimental plan.
- The possibility of measuring the interactions involved in the factorial experiment which is not possible in simple experiments.
- If the interactions between the transactions are not important, then choosing the effects of all factors at the same time by conducting a simple single experiment.

5.2 Main Effects and Interactions

Main and minor effects: It is the main effect of the factor (Main effect a factor) that is the expression in the response as a result of a change in the level of the factor, and these effects are called the main because they receive the most attention in the experiment. He also defined the (simple effect factor) a factor whose race is the difference in response between the level of a factor at a certain level of a wage factor, the (interactions). The interaction or intervention between factors was defined as a response to a factor under the influence of different levels of the factor or other factors. The effect of one factor on another is that the interaction is induced when the response changes or the response to one or more changes with a full effect, as for other factors in the study, one of which changes the levels of the other factor or the levels of the other factors). In their influence on each other, and therefore the simple effects of the factor differ and depend on the improvement of the level of the factor or other factors involved in the interaction, and in such cases, the use of intermediate experiments (with one factor leads to obtaining deficient results, but if the interaction is not significant). The empirical factors are independent of each other, and the effects of (one factor) are equal to the levels of the other factor or factors. In such a case, the minor effects equal the main effect.

5.3 Replications

Replication is important because it affects well on the course of the experiment, and the issue of determining the number of repetitions in the experiment is possible if we take into account the type of experiment, is it a field or an experiment. If the experiment is a laboratory, this means that we can increase the number of repetitions to the largest possible number without any difficulties and happiness, we choose a number within the range 8-12 repetitions, but if the experiment is field, this means that we cannot increase the number of repetitions to the largest possible number, but we choose the number of possibilities Available and usually we choose when within the range 2-8 repetition (Rayner, 1967).

5.4 Factorial Experiment 2^k

The general formula is factorial experiments 2^k with (r) repetitions, estimating the effects and total squares in general (Saha & Das, 1971)
Estimate parameters =(contrast) / (2^k. r)
Impact estimation=Parameter estimate ×2
Sum of squares SSQ = $(\text{Contrast})^2$ / (2^k. r)
Var(Constraints = (σ^2. 2^k. r)
Var (Estimate parameters = (var {Contrast} / (2^k. r)^2 = σ^2 / (2^k. r)
Var (Estimate Impact = ($2^2 \sigma^2$ / (2^k. r) = σ^2 / (2^{k-2}. r)
Confidence limits for parameters or effects when estimating (σ^2) lik this:
$I_{1-\alpha}$(Parameter) = Parameter estimate $\pm$ s.t$(f)_{(1-\alpha/2)}$ /$\sqrt{2^k . r}$
$I_{1-\alpha}$ (effect) = Effect estimates $\pm$2. s. t $(f)_{(1-\alpha/2)}$ / $\sqrt{2^k . r}$
And based $t(f)_{(1-\alpha/2)}$ The tow-sided test in the distribution, t-with f degrees of freedom.

6- Fractional Factorial Experiment

The most important difficulties facing the researcher who wants to use factorial experiments is the significant increase in the number of the treatments as the number of factors and the number of their levels increases, and thus the experiment requires a huge amount of experimental material and a high flavor. For example, a factorial experiment of (2^6) requires (64) experimental units for one repetition and this experiment allocates (6) degrees and to the degree of freedom of quantitative interactions and freedom for main effects and (15) degrees of freedom for interactions Bin higher. The number of interactions is calculated on the degree (p) of a number (n) of the following equation factors(Dean & Lewis, 1984; Kerr & Churchill, 2001; Kuhfeld et al., 1994). [Agarwal, M.L. [2000]] & [Hossam Othman Hassan Al-Khatib, 2012**]**

$$\frac{n!}{p!\,(n-p)!} \qquad \qquad . . . (1)$$

Where:
n: number of factors.
p: The degree of interaction
when (n) increases, the number of interactions with three or more factors increases, and although higher order interactions can be estimated, this does not mean that interactions are significant, and therefore we can assume that higher order interactions are not significant or It is equal to zero, and therefore it can be ignored by including it in the experimental plan and in general the large increase in the number of treatments requires a huge amount of units of material (experimental and at a high cost, which is inconsistent with the objective of the experiment which is to obtain accurate information at the lowest possible cost, And the scientist (1945 Finney) was able to solve the problem of increasing the number of processors when the number of factors increases, by modifying the designs of the full factorial experiments so that the experiment can be conducted on only a part of the complete repetition, if we assume that the announcement of higher ranks is unimportant and can be ignored These designs are called (fractional factorial experiment designs) and the necessary information about important factor effects are obtained.

6.1 Fractional Replication

Fractional Replication which is partial repetition, that it is to perform the factorial experiment on only a portion of the complete repetition with obtaining information about the main effects in the interactions of lower degrees (Low Order Interactions (Wilk & Kempthorne, 1956). We deny that it is possible to obtain on partial information, this is done by using a part of the information using a design called (Fractional Design), which leads to reducing the size of the total experiment and obtaining important information related to the factors.

6.2 Fractional Replication of A Factorial Experiment 2^k

To clarify the basic principles of forming partial Replicate in Experiments(2), which are considered the simplest types of high partial experiments, and the complete repetition of these experiments allows the researcher to estimate (k) a major effect And (2^k) a Block and triple interaction and ... and finally one interaction between (K) factor, and in it show how to obtain the necessary information about the main effects and interactions of lower degrees using a number of treatments less than (2^k) where between (1945, Finney) when there is a factorial experience (2^k) and a large (k) is used for the repetition row, the experiment is called (1/2) repetition of the reproductive experiment(2^k), and also when using a quarter of the auditory repetition of the experiment (1/4) repetition of the experiment is high (2^k). Using (1/8) iteration of the experiment is factorial (2^k) (Kuhfeld et al., 1994; Mahdi, Nassar, & Almsafir, 2019). Suppose we have a factorial experiment 2^4 and you want to use half iteration, so we symbolized the facters with the symbol (A, B, C, D) and took the nwgative sign (-) to represout the low level or normal state of the factors and a positive sign (+) to represeut the hign or varible leval in the Case of the Worker as Showh in the following table:

Table 1

Represents the table of signs for factorial experiment 2^4

effects	(1)	a	b	c	d	ab	Ac	ad	bc	bd	cd	abc	abd	acd	acd	abcd
A	-	+	-	-	-	+	+	+	-	-	-	+	+	+	-	+
B	-	-	+	-	-	+	-	-	+	+	-	+	+	-	+	+
C	-	-	-	+	-	-	+	-	+	-	+	+	-	+	+	+
D	-	-	-	-	+	-	-	+	-	+	+	-	+	+	+	+
AB	+	-	-	+	+	+	-	-	-	-	+	+	+	-	-	+
AC	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
AD	+	-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
BC	+	+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
BD	+	+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
CD	+	+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
ABC	-	+	+	+	-	-	-	+	-	+	+	+	-	-	-	+
ABD	-	+	+	-	+	-	+	-	+	-	+	-	+	-	-	+
ACD	-	+	-	+	+	+	-	-	+	+	-	-	-	+	-	+
BCD	-	-	+	+	+	+	+	+	-	-	-	-	-	-	+	+
ABCD	+	-	-	-	-	+	+	+	+	+	+	-	-	-	-	+

And if we assume that ABCD is used as a (Defining contrast) we will get the following two Block bd:

Block1	(1)	ab	ac	ad	bc	cd	bd	abcd
Block2	a	b	c	d	abc	abd	acd	bcd

Where the combination between them is the ABCD interaction, and any of the two Block is half the frequency of the factorial experiment (2^4) and the first Block included the treatments that have a sign (+) for the ABCD interaction in the signs table (1), and the second Block included the treatments that have a sign (-) for the ABCD reaction, that is, the first Block contains the processors that have an even number of letters(Wold, 1978).

The Conditions of the model are:

$$\sum_{i=0}^{a} A_i = \sum_{j=0}^{b} B_j = \sum_{i,j=0}^{a,b} AB_{ij} = \sum_{j,k=0}^{j,k} CB_{jk} = \sum_{k=0}^{c} C_k = \cdots = \sum_{i,j,k=0}^{abc} ABC_{ijk} = 0$$

Which:

$A_1=-A_0$, $B_1=-B_0$, $AB_{11}=-AB_{10}=AB_{00}$, $C_1=-C_0$, ... , (like this)
$ABC_{ooo}=-ABC_{100}=-ABC_{010}=ABC_{110}=-ABC_{001}=ABC_{101}=ABC_{011}=-ABC_{111}$
Effects on experience (that give different responses when a factor changes from low to high level).

$A=2A_1$, $B=2B_1$, $AB=2AB_{11}$, $C=2C_1$, ... , $ABC=2ABC_{111}$
For example in a factorial experiment of 2^8 the replicates
(1) , a , b , ab , c , ac , bc , abc

The estimation of the influence of factors is measured according to the following :

[I] = [+(1) + a + b + ab + c + ac + bc + abc]
[A] = [-(1) + a – b + ab – c + ac – bc + abc]
[B] = [-(1) – a + b + ab – c – ac + bc +abc]
[AB] = [+(1) – a – b + ab + c – ac – bc + abc]
[C] = [-(1) – a – b – ab + c + ac + bc + +abc]
[AC] = [+(1) – a + b – ab – c + ac – bc + abc]
[BC] = [+(1) + a – b – ab – c – ac + bc + abc]
[ABC] = [-(1) + a + b – ab + c – ac – bc + abc]
In matrix form

$$\begin{bmatrix} I \\ A \\ B \\ AB \\ C \\ AC \\ BC \\ ABC \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} (1) \\ a \\ b \\ ab \\ c \\ ac \\ bc \\ abc \end{bmatrix}$$

Estimate parameters When k=3 is

$$\hat{\mu} = \frac{[I]}{2^k . r} \ , \ \hat{A}_1 = \frac{[A]}{2^k . r} \ \hat{B}_1 = \frac{[B]}{2^k . r} \ , ..., \widehat{ABC}_{111} = \frac{[ABC]}{2^K . r} \qquad ...(4)$$

Sum of squares to experience:

$$SSQ_A = \frac{[A]^2}{2^k . r} \ , \ SSQ_B = \frac{[B]^2}{2^k . r} \ , SSQ_{ABC} = \frac{[ABC]^2}{2^k . r} \qquad ...(5)$$

To Contrasts for factor A:

Var{[A]} = var {-(1) + a – b + ab – c + ac – bc + abc} = $2^k . r . \sigma^2$ ... (6)
When k=3
Var(Mahdi et al.) = var{$\sum_{v=1}^{1} Y_{110v}$}= r . σ^2 . . . (7)
Var{$\hat{A}_1$} =Var{[A]/(2^k . r)} = σ^2/(2^k . r) . . . (8)
Var{$\hat{A}$} = Var{$2\hat{A}_1$}= σ^2/(2^{k-2} . r) . . . (9)

6.3 Statistical Inferences for A Fractional Factorial Experiment 2^k

To find the sum of squares for a factor in a Bold Fractionel factorial experiment (2^k): There is no difference in obtaining the sum of the squares of the effects using the same methods in the full factorial experiments. For example, the reference table can be used to find the sum of squares, for example, for a factorial experiment (2^k) the sum of squares SS for factor(A) is calculated from the following formula (Mahdi et al., 2019).

$$SS_A = \frac{(A\ Comparison)^2}{n \times 2^{k-1}} \quad \ldots (10)$$

Where:

n: is the number of times the experiment is repeated

As for the main effect of factor (A), it is calculated from the following relationship:

$$A\ effect = \frac{(A\ \text{Comparison})}{2^{k-2} \times n} \quad \ldots (11)$$

Where:
n: several times the factorial experiment (2^k) is repeated in half.
Statistical analysis of a factorial experiment (2^4)

Table 2
Table of analysis of variance for a factorial experiment 2^4 with half Replication

s.o.v	**Df**
A(BCD)	1
B(ACD)	1
C(ABD)	1
D(ABC)	1
AB(CD)	1
AC(BD)	1
AD(BC)	1
Error	8(n-1)
Total	8 n-1

It is clear from the analysis of variance table that the main effects are synonymous with the third-order interactions and the second-order interactions are associated with other interactions of the same order, i. e. a reciprocal offset. For another, for example, let us have a factorial experiment of (2^5) in (A, B, C, D, E) then the analysis of variance table for a factorial experiment 2^5 with half Replicate is shown in Table (3)

Table 3
Table of analysis of variance for a factorial experiment of 2^5 with half repetition

S. O. V	**DF**
Main effects or (fourth order interactions)	5
Second-order reactions (or third-order reactions)	10
Error	16(n-1)

Assuming that the interactions of the third order and above can be ignored, the partial Replicate provides information for testing the main effects and the interactions of the second order. We deny [22] that design (2^{3-1}) is considered simple and in practice, as is the case for designs (2^{4-1}) and (2^{5-1}) because they do not leave sufficient degrees of freedom for the experimental plan, and therefore it may not be used unless an estimate of experimental error is available from previous experiments.

7- Design Efficiency

Efficiency is a measure of design quality. Common measures of efficiency (N_D x p) and orthogonal design matrix (X) are based on the information matrix (X'X). The covariance matrix of the vector estimates of parameter (β)in least squares analysis is proportional to $(X'X)^{-1}$. The variance of ($\widehat{\beta_1}$)is proportional to the element $(X'X)^{-1}$. (Xii.) An effective design will have a small variance matrix, and the eigenvalues of $(X'X)^{-1}$ provider and Scale-Scale Scales Two common efficiency measures are based on the idea of mean variance or mean eigenvalue. **A-Efficiency** is a function of the arithmetic mean of the variances, which is given by trace $((X'X)^{-1})$. (The trace is the sum of the diagonal elements of $(X'X)^{-1}$, and it is the sum of the variances, and it is also the sum of the eigenvalues of $(X'X)^{-1}$. **D-Efficiency** is a function of the geometric mean of the eigenvalues, and it is given by $|(X'X)^{-1}|^{1/p}$. (Determined $|(X'X)^{-1}|$, is the product of the eigenvalues to $(X'X)^{-1}$, and the root P^{th} is the geometric mean). (A) third common measure of efficiency is **G-efficiency**, based in and the maximum standard error of the candidate group prediction. All three parameters are convex functions of eigenvalues $(X'X)^{-1}$ and are therefore usually highly correlated (Rayner, 1967; Saha & Das, 1971).

For all three criteria, if there is a balanced and orthogonal design, then it is efficient, its tendency towards balance and orthogonality increases. Assuming orthogonally encoded(X):

• The design is parallel and perpendicular when it is $(X'X)^{-1}$ diagonal
• The design is orthogonal when the sub-array to $(X'X)^{-1}$, excluding the row and column intercept, is diagonal: there may be a non-zero outside of the intercept.
• The design is balanced when all non-diagonal elements are in the intersection row and column with increased efficiency,
• The absolute values of the diagonal elements become smaller and the diagonals approach $\frac{1}{ND}$.

These efficacy metrics are Scaled to range from 0 to 100:

$$\text{A-efficiency} = \frac{1}{N_D trace\,((X'X)^{-1})^{\frac{1}{p}})} \times 100 \qquad \text{. . . (12)}$$

$$\text{D-efficiency} = \frac{1}{N_D |(X'X)^{-1}|^{\frac{1}{p}}} \times 100 \qquad \text{. . . (13)}$$

$$\text{G-efficiency} = \frac{\sqrt{p/N_D}}{\sigma_M} \times 100 \qquad \text{. . . (14)}$$

8- Practical Part

The data of Abu Ghraib factories for dairy production was used to study the factors affecting the acidity of milk. The focus was on five factors that affect the acidity of milk, which are as shown in the table.

Table 4

Represents the levels of the main factors of the experiment

High level	Low level	The Worker	Neme
14	11	A	Skimmed milk
48	45	B	Inoculation temperature
44	40	C	Babysitter temperature
4	2.30	D	Incubator time
3	1	E	Fat percentage

Confounding in the factors of the experiment where the Confounding was follows:

I + A*B*C*D*E
A + B*C*D*E
B + A*C*D*E
C + A*B*D*E
D + A*B*C*E
E + A*B*C*D
A*B + C*D*E
A*C + B*D*E
A*D + B*C*E
A*E + B*C*D
B*C + A*D*E
B*D + A*C*E
B*E + A*C*D
C*D + A*B*E
C*E + A*B*D

D*E + A*B*C

Table 5
Represents the design of the experiment

	A= B*C*D*E	B= A*C*D*E	C= A*B*D*E	D= A*B*C*E	E= A*B*C*D	Replicate
1	14(+)	45(-)	40(-)	2.3(-)	1(-)	0.23678
2	14(+)	45(-)	40(-)	2.3(-)	3(+)	0.71035
3	14(+)	48(+)	40(-)	2.3(-)	1(-)	0.25257
4	11(-)	48(+)	40(-)	2.3(-)	3(+)	0.59534
5	14(+)	45(-)	44(+)	2.3(-)	1(-)	0.26046
6	11(-)	45(-)	44(+)	2.3(-)	3(+)	0.61395
7	11(-)	48(+)	44(+)	2.3(-)	1(-)	0.21829
8	14(+)	48(+)	44(+)	2.3(-)	3(+)	0.83348
9	14(+)	45(-)	40(-)	4.0(+)	1(-)	0.41180
10	11(-)	45(-)	40(-)	4.0(+)	3(+)	0.97067
11	11(-)	48(+)	40(-)	4.0(+)	1(-)	0.34513
12	14(+)	48(+)	40(-)	4.0(+)	3(+)	1.31776
13	11(-)	45(-)	44(+)	4.0(+)	1(-)	0.35591
14	14(+)	45(-)	44(+)	4.0(+)	3(+)	0.91585
15	11(-)	48(+)	44(+)	4.0(+)	1(-)	0.37964
16	11(-)	48(+)	44(+)	4.0(+)	3(+)	0.93625

Table 6
The analysis of Variance represents the main and interaction factors

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	5	2.20766	2.12421	0.424842	121.22	0.000
2-Way Interactions	6	0.07638	0.07638	0.012729	3.63	0.116
Residual Error	4	0.01402	0.01402	0.003505		
Total	15	2.29805				

Through the results of the analysis of Variance table on the Significance of the effect of the main factors and the binary interactions, which Shows the effect of these, we note that the main factor effect is generally Significant, because the Value of P-Value is less than 0.05.
As for the effect of binary reactions, in general, there is no effect of binary reactions on acidity, because the Value P-Value is greater than 0.05.

Table 7
It Represents the estimation of the effect and Coefficients on the acidity percentage

Term	Effect	Coef.	SE Coef.	T	P
Constant		0.62500	0.01480	42.23	0.000
A	0.19517	0.09758	0.01872	5.21	0.006
B	0.06908	0.03454	0.01552	2.23	0.090
C	0.08869	0.04435	0.01552	2.86	0.046
D	0.36849	0.18424	0.01552	11.87	0.000
E	0.63485	0.31743	0.01480	21.45	0.000
B*C	-0.02502	-0.01251	0.01480	-0.85	0.446
B*D	0.00075	0.00037	0.01480	0.03	0.981
B*E	-0.01118	-0.00559	0.01552	-0.36	0.737
C*D	0.00711	0.00356	0.01480	0.24	0.822
C*E	-0.00090	-0.00045	0.01552	-0.03	0.978
D*E	0.13981	0.06990	0.01552	4.50	0.011

S = 0.0591996 R-Sq = 99.39% R-Sq(adj) = 97.71%

from the results of effect estimation table and the treatments on the acidity Percentage on the Significance of the effect of the main factors and the binary interactions, Which Shows the effect of these factors on the acidity in the experiments, we note that the effect of the main factors (A, C, D, E) is Significant because the Value of a is less than 0.05.

As for the effect of binary reactions, we note that reaction of (D*E) affects the acidity, because the Value of P-Value is less than 0.05.
It is also noted the Standard deviation value of the model is 0.059 and the Value of the Coefficient of determination is 99.39 %, Which indicates the Strength of the model, as the mathematical model for it Will be:

$$y = B_0 + \sum_{i=1}^{n} BiXi + \sum_{i=1}^{n} BijXiXj$$

Since the estimated equation of model is:

y = 0.625 + 0.09758*A + 0.04435*C+
0.18424*D+0.31743*E+0.0699*(D*E)

Table 8
Shows the estimated Values of acidity through the estimation equation

Obs.	Std Order	T1	Fit	SE Fit	Residual	St Resid
1	1	0.23678	0.14694	0.05356	0.08984	1.14
2	2	0.71035	0.78180	0.05356	-0.07145	-0.91
3	3	0.25257	0.22584	0.05671	0.02673	0.35
4	4	0.59534	0.62623	0.06254	-0.03089	-0.43
5	5	0.26046	0.24546	0.05671	0.01500	0.20
6	6	0.61395	0.64584	0.06254	-0.03189	-0.45
7	7	0.21829	0.08989	0.05671	0.12840	1.68
8	8	0.83348	0.95921	0.06254	-0.12573	-1.76
9	9	0.41180	0.52525	0.05671	-0.11345	-1.49
10	10	0.97067	0.92564	0.06254	0.04503	0.63
11	11	0.34513	0.36969	0.05671	-0.02456	-0.32
12	12	1.31776	1.23901	0.06254	0.07875	1.10
13	13	0.35591	0.38930	0.05671	-0.03339	-0.44
14	14	1.35894	1.25862	0.06254	0.10032	1.40
15	15	0.37964	0.46820	0.05356	-0.08856	-1.13
16	16	1.13892	1.10306	0.05356	0.03586	0.46

Table (8) Shows the estimated Values of the equation of the mathematical model of the experiment Where the Standard error of the estimated Values and the residuals Were Calculated.

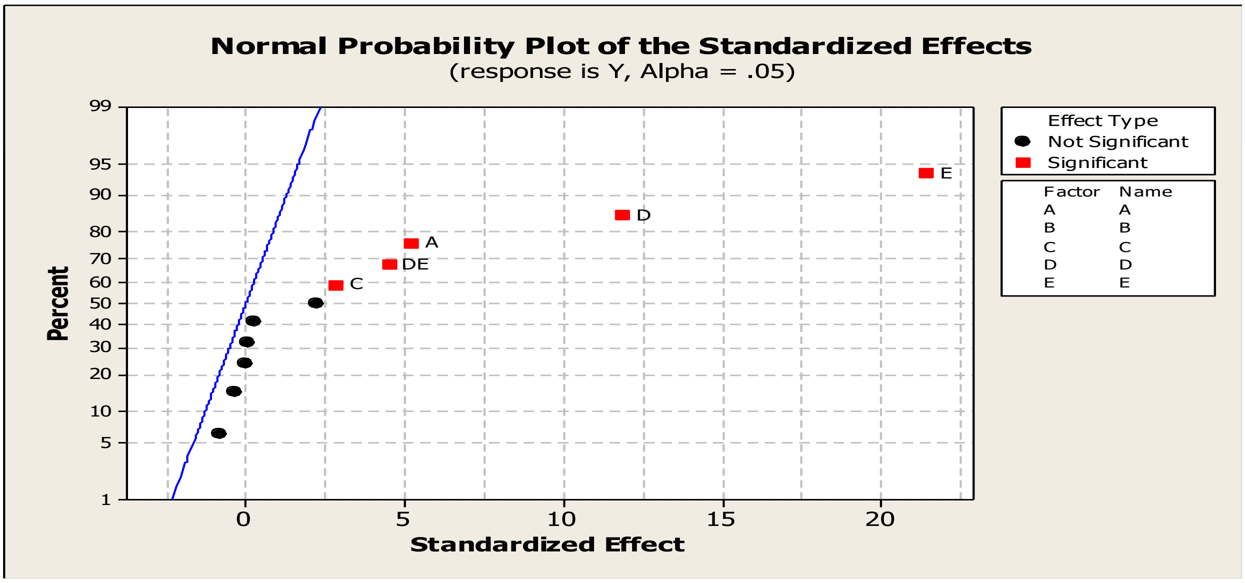

Figure 1. Normal probability plot of effect type

Through figure no (1) Which Shows the test Normal Probability for the Significance of the effect We notice the Significance of the effects (A,C,D,E,D*E) .

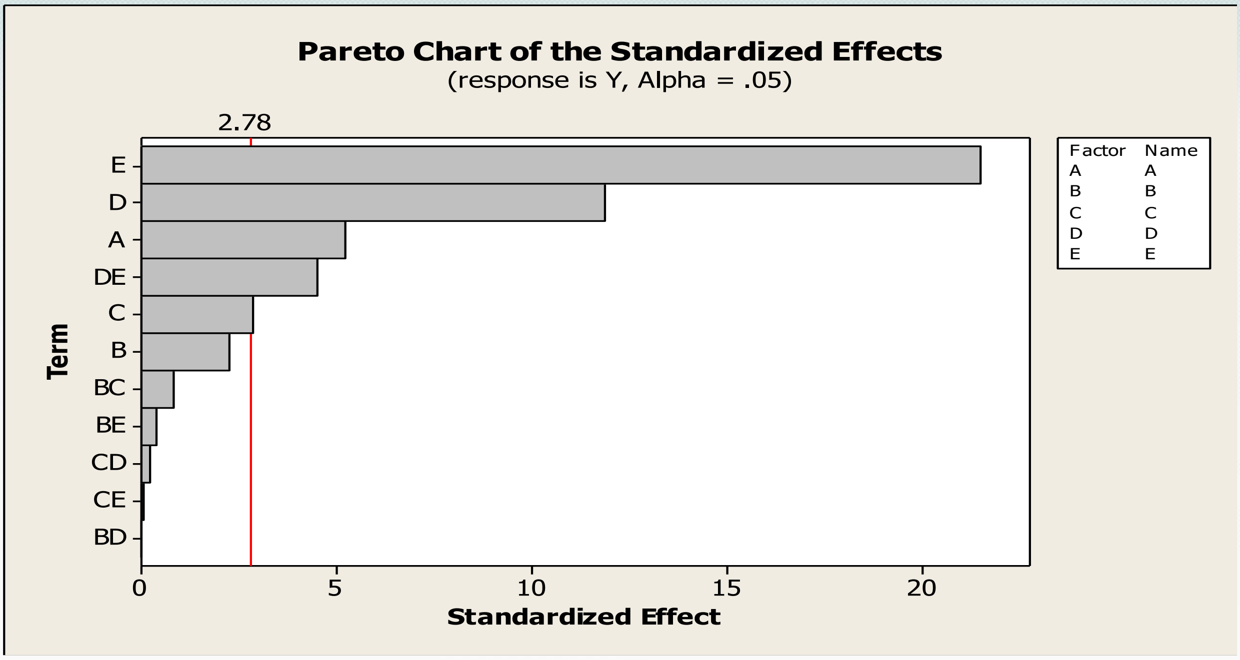

Figure 2. Effects Pareto for response

Through figure no (2) Which Shows the test Pareto Chart for the Significance of the effect We note the Significance of the effects (A, C, D, E, D*E) as the threshold limit for the test Pareto for these factors is (2.78) as the level of Significance 0.05.

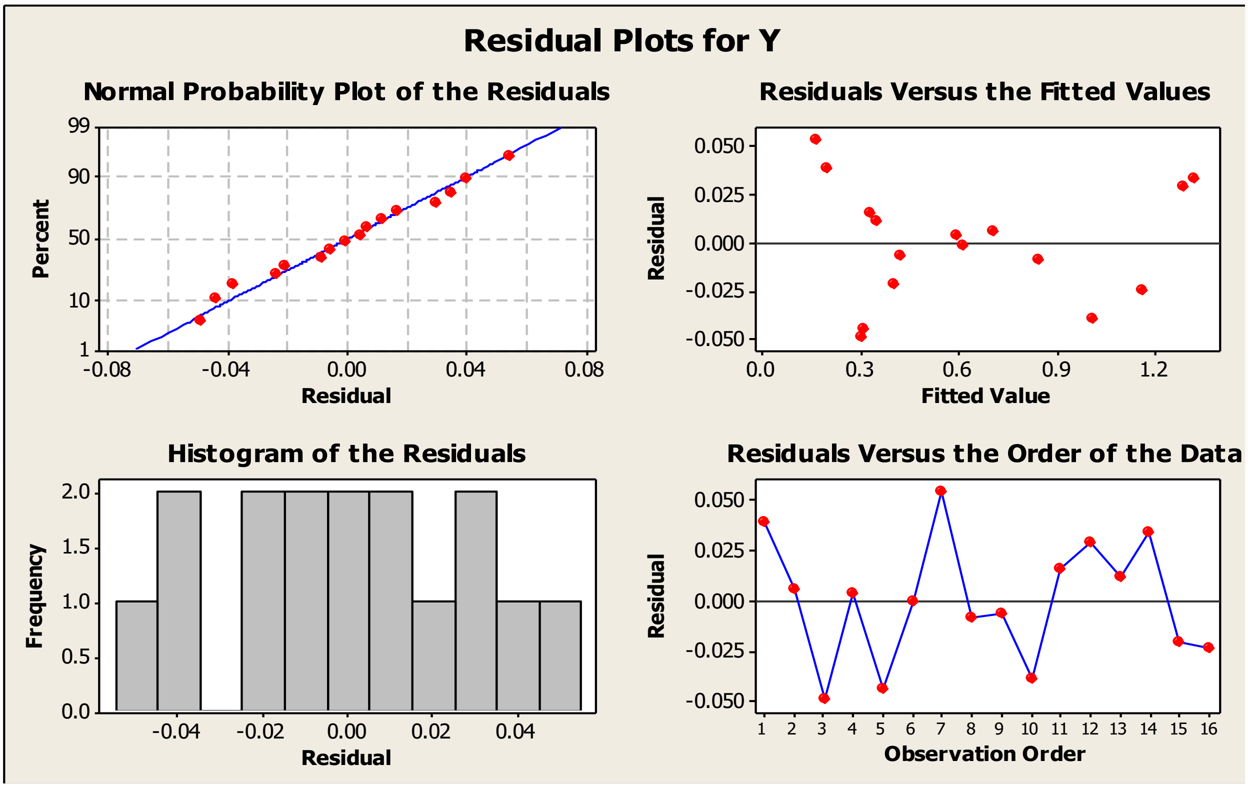

Figure 3. Model adequacy checking plots

Figure (3) represent the residuals versus the fitted values y. This plot reveals that there is not any inequality among variances and the relationship between the size of residuals and fitted values. Accordingly, there are not any severe problems in model adequacy checking that make the analysis of the model erratic and unreliable.

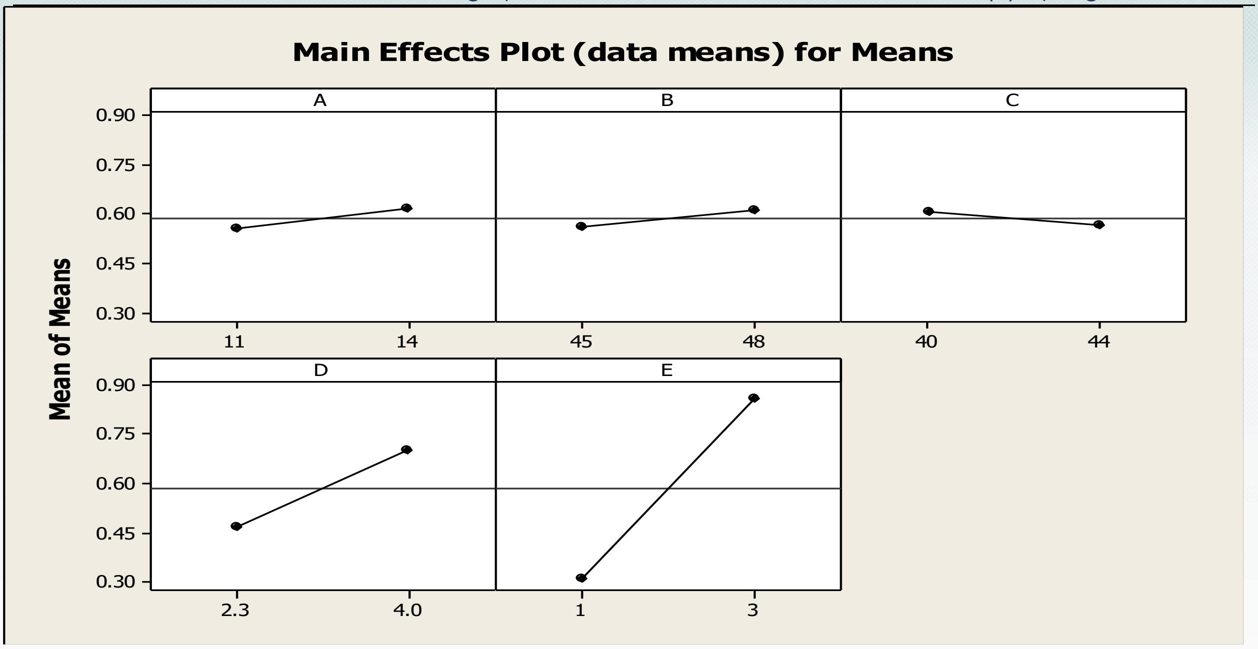

Figure 4. Main Effect Plot

Figure (4) represent when fat percentage rose from low level to high level, the pH of yogurt increased. In addition, it is clear that increasing the incubation time reduces the pH of yogurt.

Table 9

Response Optimization

Parameters

Goal	Lower	Target	Upper	Weight	Import
get	0.65	0.68	0.7	1	

Global Solution

A = 11.0000
B = 47.9750
C = 43.9624
D = 3.3568
E = 2.1261

Predicted Responses

Y = 0.68, desirability = 1

Through Table no (9) Shows the ideal Value of the main factors in this experiment, which achiere the ideal acidity ratio, which is 68%, Where for factors A=11 and factor B=47.97 and factor C=43.96 and factor D=3.35 and factor E=2.12.

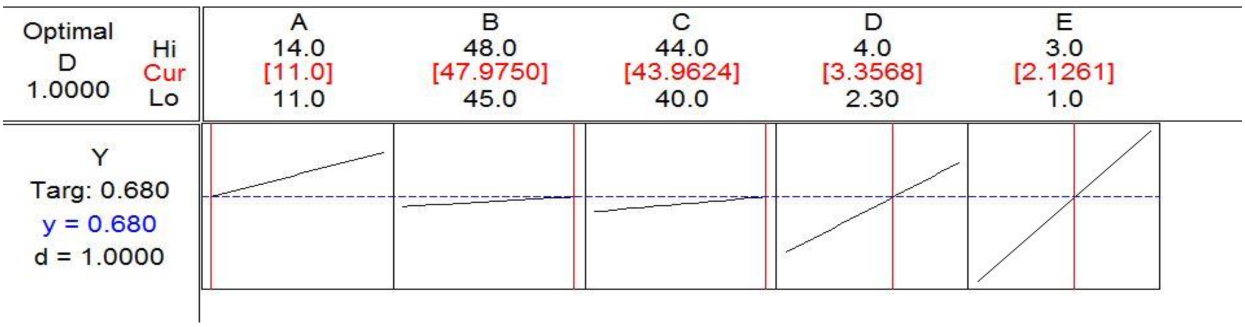

Figure 5. shows optimal D

Conclusions:

Based on the results obtained using the method of partial factorial experiments in a randomized complete block design, we find the following:

1- The factors of incubation Skimmed milk (A), Inoculation temperature (C), Incubator time (D), Fat percentage(E) and the interaction effect (DE), correspondingly. The pH values of fermented milk were factors A=11 and factor B=47.97 and factor C=43.96 and factor D=3.35 and factor E=2.12. These results indicated that the significant controllable factors were incubation time and fat percentage.
2- For reaching to the optimal range and target of the pH values, the high level of incubation time and the low level of fat percentage should be considered.

Recommendation

1. Future research directions can be study on another design can be more efficiency to estimate the parameters in model.
2. By application of fractional factorial design concept to yogurt production process, considerable improvements can be gained at a faster rate.

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